PROBLEM 13.1

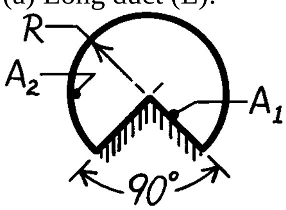
KNOWN: Various geometric shapes involving two areas A_1 and A_2.

FIND: Shape factors, F_{12} and F_{21}, for each configuration.

ASSUMPTIONS: Surfaces are diffuse.

ANALYSIS: The analysis is not to make use of tables or charts. The approach involves use of the reciprocity relation, Eq. 13.3, and summation rule, Eq. 13.4. Recognize that reciprocity applies to two surfaces; summation applies to an enclosure. Certain shape factors will be identified by inspection. Note L is the length normal to page.

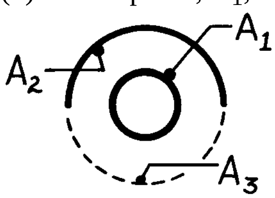
(a) Long duct (L):



By inspection, $F_{12} = 1.0$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2\,RL}{(3/4)\cdot 2\pi RL} \times 1.0 = \frac{4}{3\pi} = 0.424$ <

(b) Small sphere, A_1, under concentric hemisphere, A_2, where $A_2 = 2A$

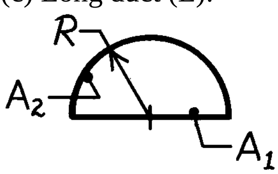


Summation rule $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{2A_1} \times 0.5 = 0.25.$ <

(c) Long duct (L):



<

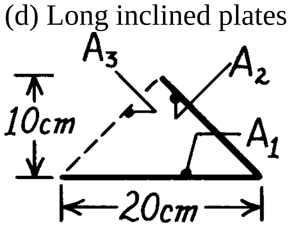
By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2RL}{\pi RL} \times 1.0 = \frac{2}{\pi} = 0.637$ <

Summation rule, $F_{22} = 1 - F_{21} = 1 - 0.64 = 0.363.$ <

By inspection,

$F_{12} = 1.0$

(d) Long inclined plates (L):

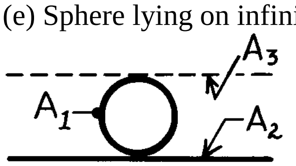


Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{20L}{10(2)^{1/2} L} \times 0.5 = 0.707.$ <

(e) Sphere lying on infinite plane



Summation rule, $F_{11} + F_{12} + F_{13} = 1$

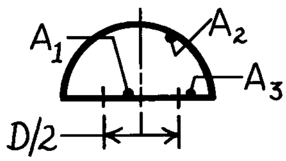
But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.5$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} \rightarrow 0$ since $A_2 \rightarrow \infty$. <

Continued …..

PROBLEM 13.1 (Cont.)

(f) Hemisphere over a disc of diameter D/2; find also F_{22} and F_{23}.



By inspection, $F_{12} = 1.0$ <

Summation rule for surface A_3 is written as

$$F_{31} + F_{32} + F_{33} = 1. \quad \text{Hence, } F_{32} = 1.0.$$

By reciprocity, $$F_{23} = \frac{A_3}{A_2} F_{32}$$

$$F_{23} = \left\{ \left[\frac{\pi D^2}{4} - \frac{\pi (D/2)^2}{4} \right] / \frac{\pi D^2}{2} \right\} 1.0 = 0.375.$$

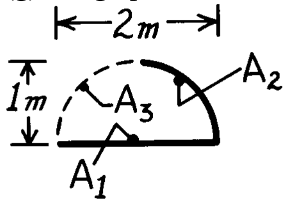
By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \left\{ \frac{\pi}{4} \left[\frac{D}{2} \right]^2 / \frac{\pi D^2}{2} \right\} \times 1.0 = 0.125.$$ <

Summation rule for A_2, $$F_{21} + F_{22} + F_{23} = 1 \text{ or}$$

$$F_{22} = 1 - F_{21} - F_{23} = 1 - 0.125 - 0.375 = 0.5.$$ <

Note that by inspection you can deduce $F_{22} = 0.5$

(g) Long open channel (L):



Summation rule for A_1

$$F_{11} + F_{12} + F_{13} = 0$$

but $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$. <

By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{(2\pi 1)/4 \times L} = \frac{4}{\pi} \times 0.50 = 0.637.$$

COMMENTS: (1) Note that the summation rule is applied to an enclosure. To complete the enclosure, it was necessary in several cases to define a third surface which was shown by dashed lines.

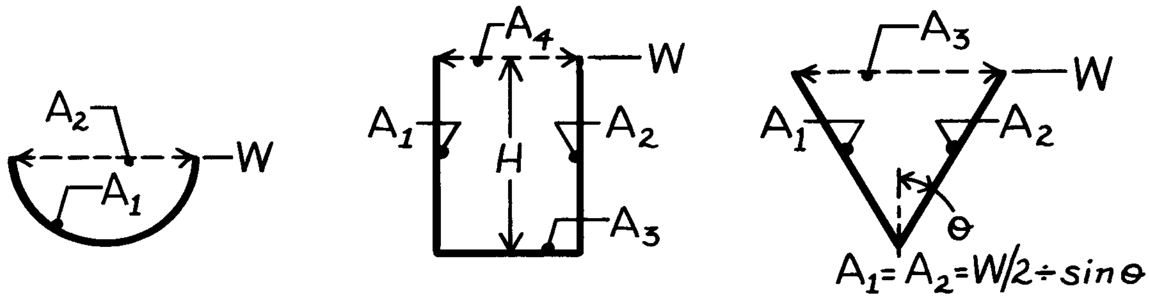
(2) Recognize that the solutions follow a systematic procedure; in many instances it is possible to deduce a shape factor by inspection.

PROBLEM 13.2

KNOWN: Geometry of semi-circular, rectangular and V grooves.

FIND: (a) View factors of grooves with respect to surroundings, (b) View factor for sides of V groove, (c) View factor for sides of rectangular groove.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surfaces, (2) Negligible end effects, “long grooves”.

ANALYSIS: (a) Consider a unit length of each groove and represent the surroundings by a hypothetical surface (dashed line).

Semi-Circular Groove:

$$F_{21} = 1; \qquad F_{12} = \frac{A_2}{A_1} F_{21} = \frac{W}{(\pi W/2)} \times 1$$

$$F_{12} = 2/\pi.$$ <

Rectangular Groove:

$$F_{4(1,2,3)} = 1; \qquad F_{(1,2,3)4} = \frac{A_4}{A_1 + A_2 + A_3} F_{4(1,2,3)} = \frac{W}{H + W + H} \times 1$$

$$F_{(1,2,3)4} = W/(W + 2H).$$ <

V Groove:

$$F_{3(1,2)} = 1; \qquad F_{(1,2)3} = \frac{A_3}{A_1 + A_2} F_{3(1,2)} = \frac{W}{\frac{W/2}{\sin\theta} + \frac{W/2}{\sin\theta}}$$

$$F_{(1,2)3} = \sin\theta.$$

(b) From Eqs. 13.3 and 13.4, $F_{12} = 1 - F_{13} = 1 - \frac{A_3}{A_1} F_{31}.$

From Symmetry, $F_{31} = 1/2.$

Hence, $F_{12} = 1 - \frac{W}{(W/2)/\sin\theta} \times \frac{1}{2}$ or $F_{12} = 1 - \sin\theta.$ <

(c) From Fig. 13.4, with X/L = H/W =2 and Y/L $\rightarrow \infty$,

$$F_{12} \approx 0.62.$$ <

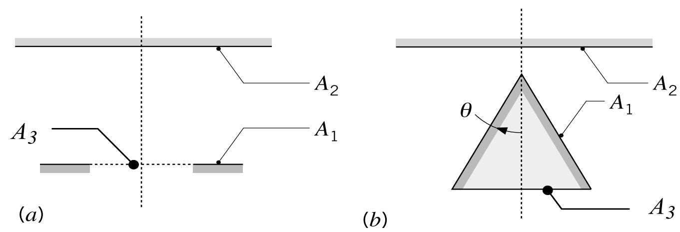
COMMENTS: (1) Note that for the V groove, $F_{13} = F_{23} = F_{(1,2)3} = \sin\theta$, (2) In part (c), Fig. 13.4 could also be used with Y/L = 2 and X/L = ∞. However, obtaining the limit of F_{ij} as X/L $\rightarrow \infty$ from the figure is somewhat uncertain.

PROBLEM 13.3

KNOWN: Two arrangements (a) circular disk and coaxial, ring shaped disk, and (b) circular disk and coaxial, right-circular cone.

FIND: Derive expressions for the view factor F_{12} for the arrangements (a) and (b) in terms of the areas A_1 and A_2, and any appropriate hypothetical surface area, as well as the view factor for coaxial parallel disks (Table 13.2, Figure 13.5). For the disk-cone arrangement, sketch the variation of F_{12} with θ for $0 \leq \theta \leq \pi/2$, and explain the key features.

SCHEMATIC:



ASSUMPTIONS: Diffuse surfaces with uniform radiosities.

ANALYSIS: (a) Define the hypothetical surface A_3, a co-planar disk inside the ring of A_1. Using the additive view factor relation, Eq. 13.5,

$$A_{(1,3)} F_{(1,3)} = A_1 F_{12} + A_3 F_{32}$$

$$F_{12} = \frac{1}{A_1}\left[A_{(1,3)} F_{(1,3)} - A_3 F_{32}\right] \quad <$$

where the parenthesis denote a composite surface. All the F_{ij} on the right-hand side can be evaluated using Fig. 13.5.

(b) Define the hypothetical surface A_3, the disk at the bottom of the cone. The radiant power leaving A_2 that is intercepted by A_1 can be expressed as

$$F_{21} = F_{23} \quad (1)$$

That is, the same power also intercepts the disk at the bottom of the cone, A_3. From reciprocity,

$$A_1 F_{12} = A_2 F_{21} \quad (2)$$

and using Eq. (1),

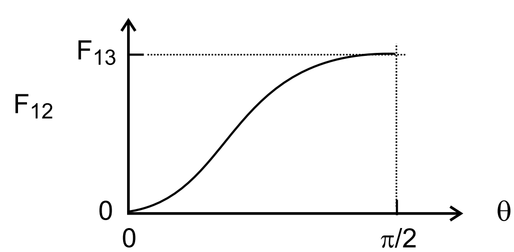
$$F_{12} = \frac{A_2}{A_1} F_{23} \quad <$$

The variation of F_{12} as a function of θ is shown below for the disk-cone arrangement. In the limit when $\theta \rightarrow \pi/2$, the cone approaches a disk of area A_3. That is,

$$F_{12}\left(\theta \rightarrow \pi/2\right) = F_{13}$$

When $\theta \rightarrow 0$, the cone area A_2 diminishes so that

$$F_{12}\left(\theta \rightarrow 0\right) = 0$$

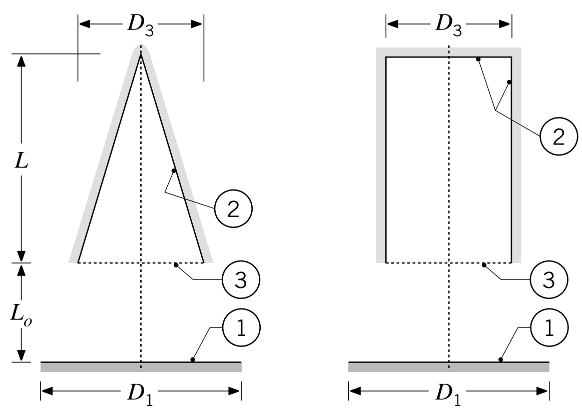


PROBLEM 13.4

KNOWN: Right circular cone and right-circular cylinder of same diameter D and length L positioned coaxially a distance L_o from the circular disk A_1; hypothetical area corresponding to the openings identified as A_3.

FIND: (a) Show that $F_{21} = (A_1/A_2)\,F_{13}$ and $F_{22} = 1 - (A_3/A_2)$, where F_{13} is the view factor between two, coaxial parallel disks (Table 13.2), for both arrangements, (b) Calculate F_{21} and F_{22} for $L = L_o$ = 50 mm and $D_1 = D_3$ = 50 mm; compare magnitudes and explain similarities and differences, and (c) Magnitudes of F_{21} and F_{22} as L increases and all other parameters remain the same; sketch and explain key features of their variation with L.

SCHEMATIC:



(a) Right-circular cone (b) Right-circular cylinder

ASSUMPTIONS: (1) Diffuse surfaces with uniform radiosities, and (2) Inner base and lateral surfaces of the cylinder treated as a single surface, A_2.

ANALYSIS: (a) For both configurations,

$$F_{13} = F_{12} \tag{1}$$

since the radiant power leaving A_1 that is intercepted by A_3 is likewise intercepted by A_2. Applying reciprocity between A_1 and A_2,

$$A_1\,F_{12} = A_2\,F_{21} \tag{2}$$

Substituting from Eq. (1), into Eq. (2), solving for F_{21}, find

$$F_{21} = (A_1/A_2)F_{12} = (A_1/A_2)F_{13}$$ <

Treating the cone and cylinder as two-surface enclosures, the summation rule for A_2 is

$$F_{22} + F_{23} = 1 \tag{3}$$

Apply reciprocity between A_2 and A_3, solve Eq. (3) to find

$$F_{22} = 1 - F_{23} = 1 - (A_3/A_2)F_{32}$$

and since $F_{32} = 1$, find

$$F_{22} = 1 - A_3/A_2$$ <

Continued …..

PROBLEM 13.4 (Cont.)

(b) For the specified values of L, L_o, D_1 and D_2, the view factors are calculated and tabulated below. Relations for the areas are:

Disk-cone: $A_1 = \pi D_1^2 / 4 \qquad A_2 = \pi D_3 / 2\left(L^2 + (D_3/2)^2\right)^{1/2} \qquad A_3 = \pi D_3^2 / 4$

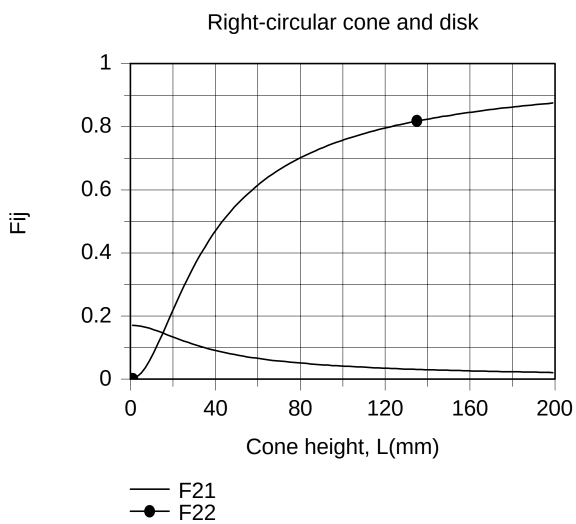
Disk-cylinder: $A_1 = \pi D_1^2 / 4 \qquad A_2 = \pi D_3^2 / 4 + \pi D_3 L \qquad A_3 = \pi D_3^2 / 4$

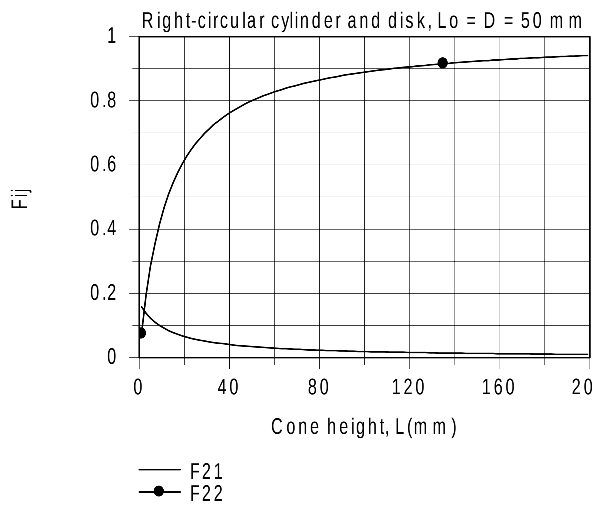
The view factor F_{13} is evaluated from Table 13.2, coaxial parallel disks (Fig. 13.5); find $F_{13} = 0.1716$.

	F_{21}	F_{22}
Disk-cone	0.0767	0.553
Disk-cylinder	0.0343	0.800

It follows that F_{21} is greater for the disk-cone (a) than for the cylinder-cone (b). That is, for (a), surface A_2 sees more of A_1 and less of itself than for (b). Notice that F_{22} is greater for (b) than (a); this is a consequence of $A_{2,b} > A_{2,a}$.

(c) Using the foregoing equations in the IHT workspace, the variation of the view factors F_{21} and F_{22} with L were calculated and are graphed below.





Note that for both configurations, when L = 0, find that $F_{21} = F_{13} = 0.1716$, the value obtained for coaxial parallel disks. As L increases, find that $F_{22} \rightarrow 1$; that is, the interior of both the cone and cylinder see mostly each other. Notice that the changes in both F_{21} and F_{22} with increasing L are greater for the disk-cylinder; F_{21} decreases while F_{22} increases.

COMMENTS: From the results of part (b), why isn't the sum of F_{21} and F_{22} equal to unity?

PROBLEM 13.5

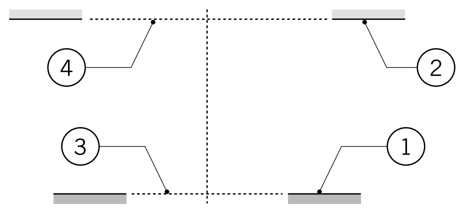
KNOWN: Two parallel, coaxial, ring-shaped disks.

FIND: Show that the view factor F_{12} can be expressed as

$$F_{12} = \frac{1}{A_1}\left\{A_{(1,3)}\, F_{(1,3)(2,4)} - A_3\, F_{3(2,4)} - A_4\left(F_{4(1,3)} - F_{43}\right)\right\}$$

where all the F_{ig} on the right-hand side of the equation can be evaluated from Figure 13.5 (see Table 13.2) for coaxial parallel disks.

SCHEMATIC:



ASSUMPTIONS: Diffuse surfaces with uniform radiosities.

ANALYSIS: Using the additive rule, Eq. 13.5, where the parenthesis denote a composite surface,

$$F_{1(2,4)} = F_{12} + F_{14}$$

$$F_{12} = F_{1(2,4)} - F_{14} \qquad (1)$$

Relation for $F_{1(2,4)}$: Using the additive rule

$$A_{(1,3)}\, F_{(1,3)(2,4)} = A_1\, F_{1(2,4)} + A_3\, F_{3(2,4)} \qquad (2)$$

where the check mark denotes a F_{ij} that can be evaluated using Fig. 13.5 for coaxial parallel disks.

Relation for F_{14}: Apply reciprocity

$$A_1\, F_{14} = A_4\, F_{41} \qquad (3)$$

and using the additive rule involving F_{41},

$$A_1\, F_{14} = A_4\left[F_{4(1,3)} - F_{43}\right] \qquad (4)$$

Relation for F_{12}: Substituting Eqs. (2) and (4) into Eq. (1),

$$F_{12} = \frac{1}{A_1}\left\{A_{(1,3)}\, F_{(1,3)(2,4)} - A_3\, F_{3(2,4)} - A_4\left(F_{4(1,3)} - F_{43}\right)\right\}$$ <

COMMENTS: (1) The F_{ij} on the right-hand side can be evaluated using Fig. 13.5.

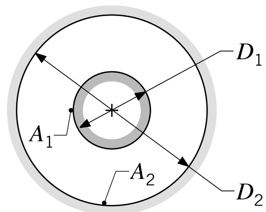
(2) To check the validity of the result, substitute numerical values and test the behavior at special limits. For example, as A_3, $A_4 \to 0$, the expression reduces to the identity $F_{12} \equiv F_{12}$.

PROBLEM 13.6

KNOWN: Long concentric cylinders with diameters D_1 and D_2 and surface areas A_1 and A_2.

FIND: (a) The view factor F_{12} and (b) Expressions for the view factors F_{22} and F_{21} in terms of the cylinder diameters.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surfaces with uniform radiosities and (2) Cylinders are infinitely long such that A_1 and A_2 form an enclosure.

ANALYSIS: (a) *View factor* F_{12}. Since the infinitely long cylinders form an enclosure with surfaces A_1 and A_2, from the summation rule on A_1, Eq. 13.4,

$$F_{11} + F_{12} = 1 \qquad (1)$$

and since A_1 doesn't see itself, $F_{11} = 0$, giving

$$F_{12} = 1 \qquad < (2)$$

That is, the inner surface views only the outer surface.

(b) *View factors* F_{22} *and* F_{21}. Applying reciprocity between A_1 and A_2, Eq. 13.3, and substituting from Eq. (2),

$$A_1 F_{12} = A_2 F_{21} \qquad (3)$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D_1 L}{\pi D_2 L} \times 1 = \frac{D_1}{D_2} \qquad < (4)$$

From the summation rule on A_2, and substituting from Eq. (4),

$$F_{21} + F_{22} = 1$$

$$F_{22} = 1 - F_{21} = 1 - \frac{D_1}{D_2} \qquad <$$

PROBLEM 13.7

KNOWN: Right-circular cylinder of diameter D, length L and the areas A_1, A_2, and A_3 representing the base, inner lateral and top surfaces, respectively.

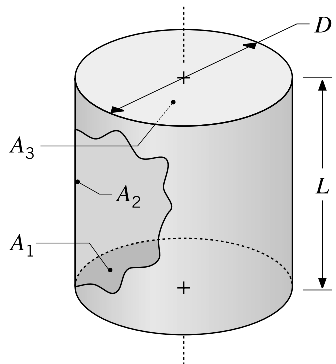
FIND: (a) Show that the view factor between the base of the cylinder and the inner lateral surface has the form

$$F_{12} = 2\,H\left[\left(1+H^2\right)^{1/2} - H\right]$$

where H = L/D, and (b) Show that the view factor for the inner lateral surface to itself has the form

$$F_{22} = 1 + H - \left(1+H^2\right)^{1/2}$$

SCHEMATIC:



ASSUMPTIONS: Diffuse surfaces with uniform radiosities.

ANALYSIS: (a) *Relation for F_{12}, base-to-inner lateral surface*. Apply the summation rule to A_1, noting that $F_{11} = 0$

$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{12} = 1 - F_{13} \qquad (1)$$

From Table 13.2, Fig. 13.5, with i = 1, *j* = 3,

$$F_{13} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(D_3 / D_1\right)^2\right]^{1/2}\right\} \qquad (2)$$

$$S = 1 + \frac{1+R_3^2}{R_1^2} = \frac{1}{R^2} + 2 = 4\,H^2 + 2 \qquad (3)$$

where $R_1 = R_3 = R = D/2L$ and H = L/D. Combining Eqs. (2) and (3) with Eq. (1), find after some manipulation

Continued …..

$$F_{12} = 1 - \frac{1}{2}\left\{4\,H^2 + 2 - \left[\left(4\,H^2 + 2\right)^2 - 4\right]^{1/2}\right\}$$

$$F_{12} = 2\,H\left[\left(1 + H^2\right)^{1/2} - H\right] \qquad (4)$$

(b) *Relation for F_{22}, inner lateral surface*. Apply summation rule on A_2, recognizing that $F_{23} = F_{21}$,

$$F_{21} + F_{22} + F_{23} = 1 \qquad\qquad F_{22} = 1 - 2\,F_{21} \qquad (5)$$

Apply reciprocity between A_1 and A_2,

$$F_{21} = \left(A_1 / A_2\right) F_{12} \qquad (6)$$

and substituting into Eq. (5), and using area expressions

$$F_{22} = 1 - 2\,\frac{A_1}{A_2}\,F_{12} = 1 - 2\,\frac{D}{4\,L}F_{12} = 1 - \frac{1}{2\,H}F_{12} \qquad (7)$$

where $A_1 = \pi D^2/4$ and $A_2 = \pi DL$.

Substituting from Eq. (4) for F_{12}, find

$$F_{22} = 1 - \frac{1}{2\,H}2\,H\left[\left(1 + H^2\right)^{1/2} - H\right] = 1 + H - \left(1 + H^2\right)^{1/2}$$ <

PROBLEM 13.8

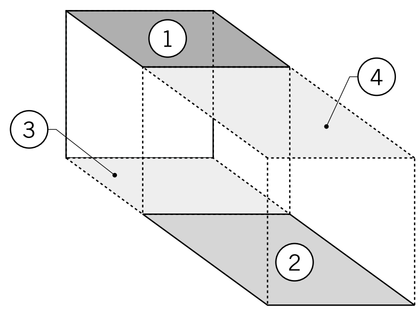
KNOWN: Arrangement of plane parallel rectangles.

FIND: Show that the view factor between A_1 and A_2 can be expressed as

$$F_{12} = \frac{1}{2\,A_1}\left[A_{(1,4)}\,F_{(1,4)(2,3)} - A_1\,F_{13} - A_4\,F_{42}\right]$$

where all F_{ij} on the right-hand side of the equation can be evaluated from Fig. 13.4 (see Table 13.2) for aligned parallel rectangles.

SCHEMATIC:



ASSUMPTIONS: Diffuse surfaces with uniform radiosity.

ANALYSIS: Using the additive rule where the parenthesis denote a composite surface,

$$A_{(1,4)}\,F^*_{(1,4)(2,3)} = A_1\,F^*_{13} + A_1\,F_{12} + A_4\,F_{43} + A_4\,F^*_{42} \qquad (1)$$

where the asterisk (*) denotes that the F_{ij} can be evaluated using the relation of Figure 13.4. Now, find suitable relation for F_{43}. By symmetry,

$$F_{43} = F_{21} \qquad (2)$$

and from reciprocity between A_1 and A_2,

$$F_{21} = \frac{A_1}{A_2}F_{12} \qquad (3)$$

Multiply Eq. (2) by A_4 and substitute Eq. (3), with $A_4 = A_2$,

$$A_4\,F_{43} = A_4\,F_{21} = A_4\,\frac{A_1}{A_2}F_{12} = A_1\,F_{12} \qquad (4)$$

Substituting for $A_4\,F_{43}$ from Eq. (4) into Eq. (1), and rearranging,

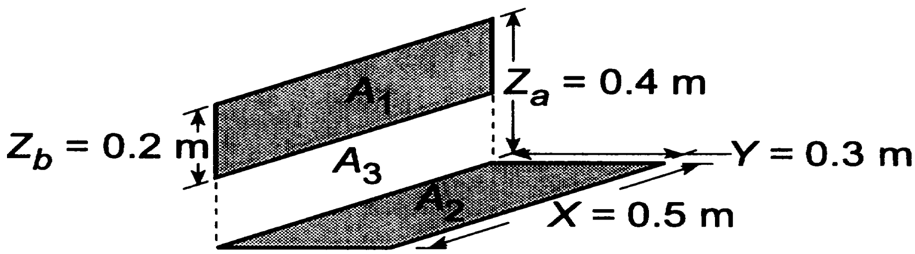
$$F_{12} = \frac{1}{2\,A_1}\left[A_{(1,4)}\,F^*_{(1,4)(2,3)} - A_1\,F^*_{13} - A_4\,F^*_{42}\right] \qquad <$$

PROBLEM 13.9

KNOWN: Two perpendicular rectangles not having a common edge.

FIND: (a) Shape factor, F_{12}, and (b) Compute and plot F_{12} as a function of Z_b for $0.05 \le Z_b \le 0.4$ m; compare results with the view factor obtained from the two-dimensional relation for perpendicular plates with a common edge, Table 13.1.

SCHEMATIC:



ASSUMPTIONS: (1) All surfaces are diffuse, (2) Plane formed by $A_1 + A_3$ is perpendicular to plane of A_2.

ANALYSIS: (a) Introducing the hypothetical surface A_3, we can write

$$F_{2(3,1)} = F_{23} + F_{21}. \quad (1)$$

Using Fig. 13.6, applicable to perpendicular rectangles with a common edge, find

$$F_{23} = 0.19: \quad \text{with } Y = 0.3, \quad X = 0.5, \quad Z = Z_a - Z_b = 0.2, \text{ and } \frac{Y}{X} = \frac{0.3}{0.5} = 0.6, \quad \frac{Z}{X} = \frac{0.2}{0.5} = 0.4$$

$$F_{2(3,1)} = 0.25: \quad \text{with } Y = 0.3, \ X = 0.5, \ Z_a = 0.4, \text{ and } \frac{Y}{X} = \frac{0.3}{0.5} = 0.6, \quad \frac{Z}{X} = \frac{0.4}{0.5} = 0.8$$

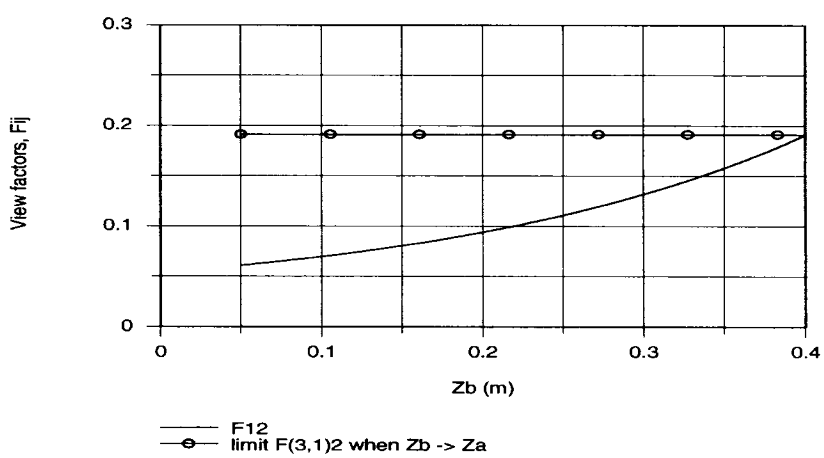
Hence from Eq. (1)

$$F_{21} = F_{2(3.1)} - F_{23} = 0.25 - 0.19 = 0.06$$

By reciprocity,

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{0.5 \times 0.3\,m^2}{0.5 \times 0.2\,m^2} \times 0.06 = 0.09 \quad (2) \quad <$$

(b) Using the *IHT Tool – View Factors* for *Perpendicular Rectangles with a Common Edge* and Eqs. (1,2) above, F_{12} was computed as a function of Z_b. Also shown on the plot below is the view factor $F_{(3,1)2}$ for the limiting case $Z_b \to Z_a$.

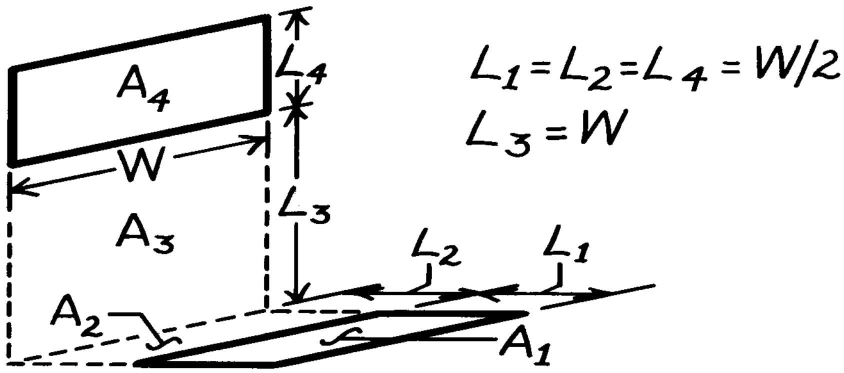


PROBLEM 13.10

KNOWN: Arrangement of perpendicular surfaces without a common edge.

FIND: (a) A relation for the view factor F_{14} and (b) The value of F_{14} for prescribed dimensions.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surfaces.

ANALYSIS: (a) To determine F_{14}, it is convenient to define the hypothetical surfaces A_2 and A_3. From Eq. 13.6,

$$(A_1 + A_2)F_{(1,2)(3,4)} = A_1\ F_{1(3,4)} + A_2\ F_{2(3,4)}$$

where $F_{(1,2)(3,4)}$ and $F_{2(3,4)}$ may be obtained from Fig. 13.6. Substituting for $A_1\ F_{1(3,4)}$ from Eq. 13.5 and combining expressions, find

$$A_1\ F_{1(3,4)} = A_1\ F_{13} + A_1\ F_{14}$$

$$F_{14} = \frac{1}{A_1}\left[(A_1 + A_2)F_{(1,2)(3,4)} - A_1\ F_{13} - A_2\ F_{2(3,4)}\right].$$

Substituting for $A_1\ F_{13}$ from Eq. 13.6, which may be expressed as

$$(A_1 + A_2)F_{(1,2)3} = A_1\ F_{13} + A_2\ F_{23}.$$

The desired relation is then

$$F_{14} = \frac{1}{A_1}\left[(A_1 + A_2)F_{(1,2)(3,4)} + A_2 F_{23} - (A_1 + A_2)F_{(1,2)3} - A_2\ F_{2(3,4)}\right].$$ <

(b) For the prescribed dimensions and using Fig. 13.6, find these view factors:

Surfaces (1,2)(3,4) $\quad (Y/X) = \frac{L_1 + L_2}{W} = 1, \quad (Z/X) = \frac{L_3 + L_4}{W} = 1.45, \quad F_{(1,2)(3,4)} = 0.22$

Surfaces 23 $\quad (Y/X) = \frac{L_2}{W} = 0.5, \quad (Z/X) = \frac{L_3}{W} = 1, \quad F_{23} = 0.28$

Surfaces (1,2)3 $\quad (Y/X) = \frac{L_1 + L_2}{W} = 1, \quad (Z/X) = \frac{L_3}{W} = 1, \quad F_{(1,2)3} = 0.20$

Surfaces 2(3,4) $\quad (Y/X) = \frac{L_2}{W} = 0.5, \quad (Z/X) = \frac{L_3 + L_4}{W} = 1.5, \quad F_{2(3,4)} = 0.31$

Using the relation above, find

$$F_{14} = \frac{1}{(WL_1)}\left[(WL_1 + WL_2)0.22 + (WL_2)0.28 - (WL_1 + WL_2)0.20 - (WL_2)0.31\right]$$

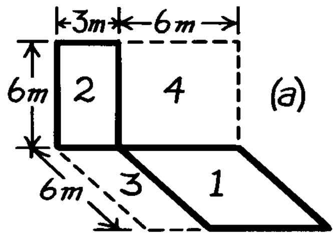
$$F_{14} = \left[2(0.22) + 1(0.28) - 2(0.20) - 1(0.31)\right] = 0.01.$$ <

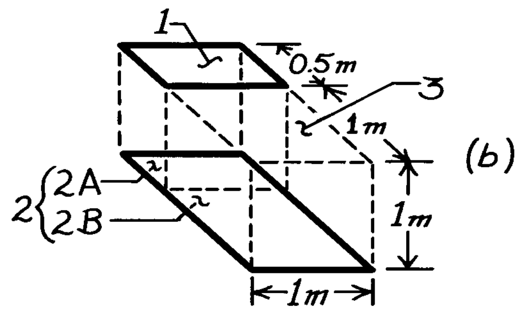
PROBLEM 13.11

KNOWN: Arrangements of rectangles.

FIND: The shape factors, F_{12}.

SCHEMATIC:





ASSUMPTIONS: (1) Diffuse surface behavior.

ANALYSIS: (a) Define the hypothetical surfaces shown in the sketch as A_3 and A_4. From the additive view factor rule, Eq. 13.6, we can write

$$A_{(1,3)} \overset{\surd}{F}_{(1,3)(2,4)} = A_1 F_{12} + A_1 \overset{\surd}{F}_{14} + A_3 \overset{\surd}{F}_{32} + A_3 F_{34} \qquad (1)$$

Note carefully which factors can be evaluated from Fig. 13.6 for perpendicular rectangles with a common edge. (See √). It follows from symmetry that

$$A_1 F_{12} = A_4 F_{43}. \qquad (2)$$

Using reciprocity,

$$A_4 F_{43} = A_3 F_{34}, \qquad \text{then} \qquad A_1 F_{12} = A_3 F_{34}. \qquad (3)$$

Solving Eq. (1) for F_{12} and substituting Eq. (3) for A_3F_{34}, find that

$$F_{12} = \frac{1}{2A_1}\left[A_{(1,3)} F_{(1,3)(2,4)} - A_1 F_{14} - A_3 F_{32}\right]. \qquad (4)$$

Evaluate the view factors from Fig. 13.6:

F_{ij}	Y/X	Z/X	F_{ij}
(1,3) (2,4)	$\frac{6}{9} = 0.67$	$\frac{6}{9} = 0.67$	0.23
14	$\frac{6}{6} = 1$	$\frac{6}{6} = 1$	0.20
32	$\frac{6}{3} = 2$	$\frac{6}{3} = 2$	0.14

Substituting numerical values into Eq. (4) yields

$$F_{12} = \frac{1}{2\times(6\times6)\text{m}^2}\left[(6\times9)\text{m}^2\times0.23 - (6\times6)\text{m}^2\times0.20 - (6\times3)\text{m}^2\times0.14\right]$$

$$F_{12} = 0.038. \qquad <$$

Continued …..

(b) Define the hypothetical surface A_3 and divide A_2 into two sections, A_{2A} and A_{2B}. From the additive view factor rule, Eq. 13.6, we can write

$$A_{1,3}\overset{\surd}{F}_{(1,3)2} = A_1F_{12} + A_3F_{3(2A)} + A_3\overset{\surd}{F}_{3(2B)}. \qquad (5)$$

Note that the view factors checked can be evaluated from Fig. 13.4 for aligned, parallel rectangles. To evaluate $F_{3(2A)}$, we first recognize a relationship involving $F_{(24)1}$ will eventually be required. Using the additive rule again,

$$A_{2A}F_{(2A)(1,3)} = A_{2A}\overset{\surd}{F}_{(2A)1} + A_{2A}F_{(2A)3}. \qquad (6)$$

Note that from symmetry considerations,

$$A_{2A}F_{(2A)(1,3)} = A_1F_{12} \qquad (7)$$

and using reciprocity, Eq. 13.3, note that

$$A_{2A}F_{2A3} = A_3F_{3(2A)}. \qquad (8)$$

Substituting for $A_3F_{3(2A)}$ from Eq. (8), Eq. (5) becomes

$$A_{(1,3)}\overset{\surd}{F}_{(1,3)2} = A_1F_{12} + A_{2A}F_{(2A)3} + A_3\overset{\surd}{F}_{3(2B)}.$$

Substituting for $A_{2A}\,F_{(2A)3}$ from Eq. (6) using also Eq. (7) for $A_{2A}\,F_{(2A)(1,3)}$ find that

$$A_{(1,3)}\overset{\surd}{F}_{(1,3)2} = A_1F_{12} + \left(A_1F_{12} - A_{2A}\overset{\surd}{F}_{(2A)1}\right) + A_3\overset{\surd}{F}_{3(2B)} \qquad (9)$$

and solving for F_{12}, noting that $A_1 = A_{2A}$ and $A_{(1,3)} = A_2$

$$F_{12} = \frac{1}{2A_1}\left[A_2\overset{\surd}{F}_{(1,3)2} + A_{2A}\overset{\surd}{F}_{(2A)1} - A_3\overset{\surd}{F}_{3(2B)}\right]. \qquad (10)$$

Evaluate the view factors from Fig. 13.4:

F_{ij}	X/L	Y/L	F_{ij}
(1,3) 2	$\frac{1}{1}=1$	$\frac{1.5}{1}=1.5$	0.25
(2A)1	$\frac{1}{1}=1$	$\frac{0.5}{1}=0.5$	0.11
3(2B)	$\frac{1}{1}=1$	$\frac{1}{1}=1$	0.20

Substituting numerical values into Eq. (10) yields

$$F_{12} = \frac{1}{2(0.5\times1)\text{m}^2}\left[(1.5\times1.0)\text{m}^2\times0.25 + (0.5\times1)\text{m}^2\times0.11 - (1\times1)\text{m}^2\times0.20\right]$$

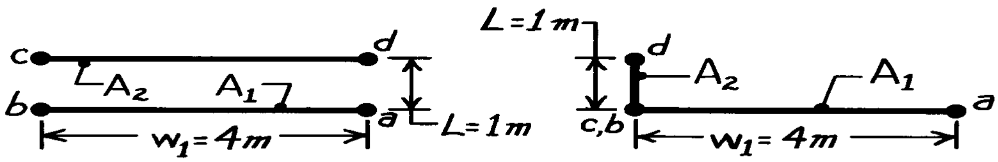
$$F_{12} = 0.23.$$ <

PROBLEM 13.12

KNOWN: Two geometrical arrangements: (a) parallel plates and (b) perpendicular plates with a common edge.

FIND: View factors using “crossed-strings” method; compare with appropriate graphs and analytical expressions.

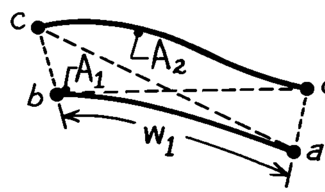
SCHEMATIC:



(a) Parallel plates (b) Perpendicular plates with common edge

ASSUMPTIONS: Plates infinite extent in direction normal to page.

ANALYSIS: The “crossed-strings” method is applicable
to surfaces of infinite extent in one direction having an obstructed view of one another.



$$F_{12} = (1/2w_1)[(ac+bd)-(ad+bc)].$$

(a) *Parallel plates:* From the schematic, the edge and diagonal distances are

$$ac = bd = \left(w_1^2 + L^2\right)^{1/2} \qquad bc = ad = L.$$

With w_1 as the width of the plate, find

$$F_{12} = \frac{1}{2w_1}\left[2\left(w_1^2+L^2\right)^{1/2} - 2(L)\right] = \frac{1}{2\times 4\ \text{m}}\left[2\left(4^2+1^2\right)^{1/2}\ \text{m} - 2(1\ \text{m})\right] = 0.781.$$ <

Using Fig. 13.4 with X/L = 4/1 = 4 and Y/L = ∞, find $F_{12} \approx 0.80$. Also, using the first relation of Table 13.1,

$$F_{ij} = \left\{\left[\left(W_i + W_j\right)^2 + 4\right]^{1/2} - \left[\left(W_i - W_j\right)^2 + 4\right]^{1/2}\right\}/2W_i$$

where $w_i = w_j = w_1$ and W = w/L = 4/1 = 4, find

$$F_{12} = \left\{\left[(4+4)^2+4\right]^{1/2} - \left[(4-4)^2+4\right]^{1/2}\right\}/2\times 4 = 0.781.$$

(b) *Perpendicular plates* with a common edge: From the schematic, the edge and diagonal distances are

$$ac = w_1 \qquad bd = L \qquad ad = \left(w_1^2 + L^2\right) \qquad bc = 0.$$

With w_1 as the width of the horizontal plates, find

$$F_{12} = (1/2w_1)\left[2(w_1+L) - \left(\left(w_1^2+L^2\right)^{1/2} + 0\right)\right]$$

$$F_{12} = (1/2\times 4\ \text{m})\left[(4+1)\text{m} - \left(\left(4^2+1^2\right)^{1/2}\ \text{m} + 0\right)\right] = 0.110.$$ <

From the third relation of Table 13.1, with $w_i = w_1$ = 4 m and w_j = L = 1 m, find

$$F_{ij} = \left\{1 + \left(w_j/w_i\right) - \left[1 + \left(w_j/w_i\right)^2\right]^{1/2}\right\}/2$$

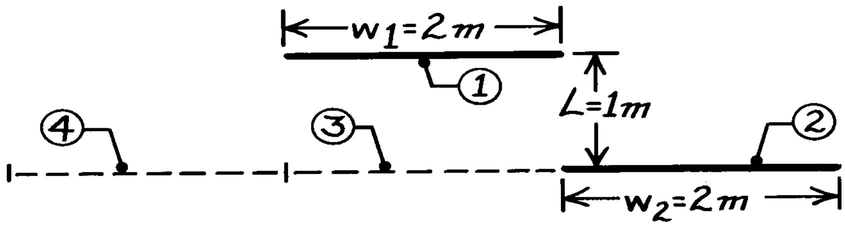
$$F_{12} = \left\{1 + (1/4) - \left[1 + (1/4)^2\right]^{1/2}\right\}/2 = 0.110.$$

PROBLEM 13.13

KNOWN: Parallel plates of infinite extent (1,2) having aligned opposite edges.

FIND: View factor F_{12} by using (a) appropriate view factor relations and results for opposing parallel plates and (b) Hottel's string method described in Problem 13.12

SCHEMATIC:



ASSUMPTIONS: (1) Parallel planes of infinite extent normal to page and (2) Diffuse surfaces with uniform radiosity.

ANALYSIS: From symmetry consideration ($F_{12} = F_{14}$) and Eq. 13.5, it follows that

$$F_{12} = (1/2)\left[F_{1(2,3,4)} - F_{13}\right]$$

where A_3 and A_4 have been defined for convenience in the analysis. Each of these view factors can be evaluated by the first relation of Table 13.1 for parallel plates with midlines connected perpendicularly.

F_{13}: $\quad W_1 = w_1/L = 2 \qquad W_2 = w_2/L = 2$

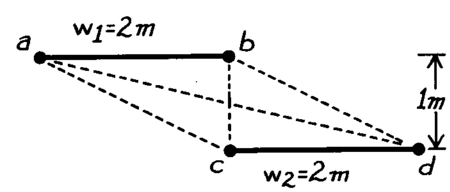
$$F_{13} = \frac{\left[(W_1+W_2)^2+4\right]^{1/2} - \left[(W_2-W_1)^2+4\right]^{1/2}}{2W_1} = \frac{\left[(2+2)^2+4\right]^{1/2} - \left[(2-2)^2+4\right]^{1/2}}{2\times 2} = 0.618$$

$F_{1(2,3,4)}$: $\quad W_1 = w_1/L = 2 \qquad W_{(2,3,4)} = 3w_2/L = 6$

$$F_{1(2,3,4)} = \frac{\left[(2+6)^2+4\right]^{1/2} - \left[(6-2)^2+4\right]^{1/2}}{2\times 2} = 0.944.$$

Hence, find $\quad F_{12} = (1/2)\left[0.944 - 0.618\right] = 0.163.$ <

(b) Using Hottel's string method,



$$F_{12} = (1/2w_1)\left[(ac+bd)-(ad+bc)\right]$$

$$ac = \left(1+4^2\right)^{1/2} = 4.123$$

$$bd = 1$$

$$ad = \left(1^2+2^2\right)^{1/2} = 2.236$$

$$bc = ad = 2.236$$

and substituting numerical values find

$$F_{12} = (1/2\times 2)\left[(4.123+1)-(2.236+2.236)\right] = 0.163.$$ <

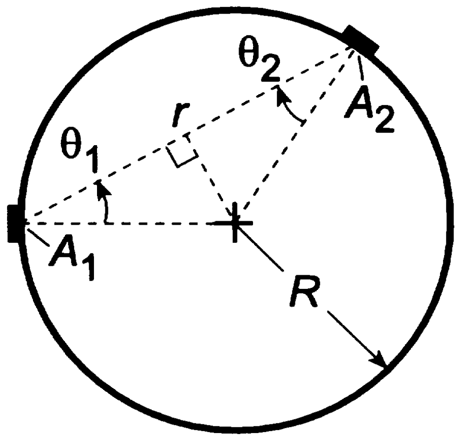
COMMENTS: Remember that Hottel's string method is applicable only to surfaces that are of infinite extent in one direction and have unobstructed views of one another.

PROBLEM 13.14

KNOWN: Two small diffuse surfaces, A_1 and A_2, on the inside of a spherical enclosure of radius R.

FIND: Expression for the view factor F_{12} in terms of A_2 and R by two methods: (a) Beginning with the expression $F_{ij} = q_{ij}/A_i J_i$ and (b) Using the view factor integral, Eq. 13.1.

SCHEMATIC:



ASSUMPTIONS: (1) Surfaces A_1 and A_2 are diffuse and (2) A_1 and $A_2 << R^2$.

ANALYSIS: (a) The view factor is defined as the fraction of radiation leaving A_i which is intercepted by surface j and, from Section 13.1.1, can be expressed as

$$F_{ij} = \frac{q_{ij}}{A_i J_i} \tag{1}$$

From Eq. 12.5, the radiation leaving intercepted by A_1 and A_2 on the spherical surface is

$$q_{1\to 2} = \left(J_1/\pi\right)\cdot A_1 \cos\theta_1 \cdot \omega_{2-1} \tag{2}$$

where the solid angle A_2 subtends with respect to A_1 is

$$\omega_{2-1} = \frac{A_{2,n}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} \tag{3}$$

From the schematic above,

$$\cos\theta_1 = \cos\theta_2 \qquad r = 2R\cos\theta_1 \tag{4,5}$$

Hence, the view factor is

$$F_{ij} = \frac{\left(J_1/\pi\right)A_1\cos\theta_1 \cdot A_2\cos\theta_2 / 4R^2\cos\theta_1}{A_1 J_1} = \frac{A_2}{4\pi R^2}$$ <

(b) The view factor integral, Eq. 13.1, for the small areas A_1 and A_2 is

$$F_{12} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1\cos\theta_2}{\pi r^2} dA_1 dA_2 = \frac{\cos\theta_1\cos\theta_2 A_2}{\pi r^2}$$

and from Eqs. (4,5) above,

$$F_{12} = \frac{A_2}{\pi R^2}$$ <

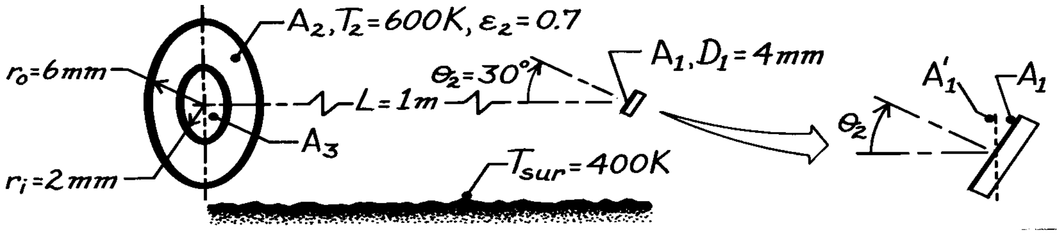
COMMENTS: Recognize the importance of the second assumption. We require that A_1, A_2, $<< R^2$ so that the areas can be considered as of differential extent, $A_1 = dA_1$, and $A_2 = dA_2$.

PROBLEM 13.15

KNOWN: Disk A_1, located coaxially, but tilted 30° of the normal, from the diffuse-gray, ring-shaped disk A_2. Surroundings at 400 K.

FIND: Irradiation on A_1, G_1, due to the radiation from A_2.

SCHEMATIC:



ASSUMPTIONS: (1) A_2 is diffuse-gray surface, (2) Uniform radiosity over A_2, (3) The surroundings are large with respect to A_1 and A_2.

ANALYSIS: The irradiation on A_1 is

$$G_1 = q_{21}/A_1 = (F_{21} \cdot J_2 A_2)/A_1 \qquad (1)$$

where J_2 is the radiosity from A_2 evaluated as

$$J_2 = \varepsilon_2 E_{b,2} + \rho_2 G_2 = \varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2)\sigma T_{sur}^4$$

$$J_2 = 0.7 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (600 \text{ K})^4 + (1-0.7) 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (400 \text{ K})^4$$

$$J_2 = 5144 + 436 = 5580 \text{ W/m}^2. \qquad (2)$$

Using the view factor relation of Eq. 13.8, evaluate view factors between A_1', the normal projection of A_1, and A_3 as

$$F_{1'3} = \frac{D_i^2}{D_i^2 + 4L^2} = \frac{(0.004 \text{ m})^2}{(0.004 \text{ m})^2 + 4(1 \text{ m})^2} = 4.00 \times 10^{-6}$$

and between A_1' and (A_2 + A_3) as

$$F_{1'(23)} = \frac{D_o^2}{D_o^2 + 4L^2} = \frac{(0.012)^2}{(0.012)^2 + 4(1 \text{ m})^2} = 3.60 \times 10^{-5}$$

giving $\qquad F_{1'2} = F_{1'(23)} - F_{1'3} = 3.60 \times 10^{-5} - 4.00 \times 10^{-6} = 3.20 \times 10^{-5}.$

From the reciprocity relation it follows that

$$F_{21'} = A_1' F_{1'2}/A_2 = (A_1 \cos\theta_1 / A_2) F_{1'2} = 3.20 \times 10^{-5} \cos\theta_1 (A_1/A_2). \qquad (3)$$

By inspection we note that all the radiation striking A_1' will also intercept A_1; that is

$$F_{21} = F_{21'}. \qquad (4)$$

Hence, substituting for Eqs. (3) and (4) for F_{21} into Eq. (1), find

$$G_1 = \left(3.20 \times 10^{-5} \cos\theta_1 (A_1/A_2) \times J_2 \times A_2\right)/A_1 = 3.20 \times 10^{-5} \cos\theta_1 \cdot J_2 \qquad (5)$$

$$G_1 = 3.20 \times 10^{-5} \cos(30°) \times 5580 \text{ W/m}^2 = 27.7 \ \mu\text{W/m}^2. \qquad <$$

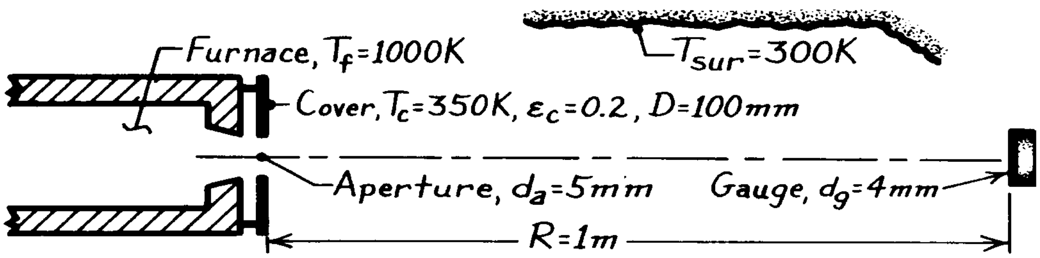
COMMENTS: (1) Note from Eq. (5) that $G_1 \sim \cos\theta_1$ such that G_1 is a maximum when A_1 is normal to disk A_2.

PROBLEM 13.16

KNOWN: Heat flux gauge positioned normal to a blackbody furnace. Cover of furnace is at 350 K while surroundings are at 300 K.

FIND: (a) Irradiation on gage, G_g, considering only emission from the furnace aperture and (b) Irradiation considering radiation from the cover *and* aperture.

SCHEMATIC:



ASSUMPTIONS: (1) Furnace aperture approximates blackbody, (2) Shield is opaque, diffuse and gray with uniform temperature, (3) Shield has uniform radiosity, (4) $A_g << R^2$, so that $\omega_{g\text{-}f} = A_g/R^2$, (5) Surroundings are large, uniform at 300 K.

ANALYSIS: (a) The irradiation on the gauge due *only* to aperture emission is

$$G_g = q_{f-g}/A_g = \left(I_{e,f} \cdot A_f \cos\theta_f \cdot \omega_{g-f}\right)/A_g = \frac{\sigma T_f^4}{\pi} \cdot A_f \cdot \frac{A_g}{R^2}/A_g$$

$$G_g = \frac{\sigma T_f^4}{\pi R^2} A_f = \frac{5.67\times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 (1000 \text{ K})^4}{\pi (1 \text{ m})^2} \times (\pi/4)(0.005 \text{ m})^2 = 354.4 \text{ mW/m}^2. \quad <$$

(b) The irradiation on the gauge due to radiation from the *aperture* (a) and *cover (c)* is

$$G_g = G_{g,a} + \frac{F_{c-g} \cdot J_c A_c}{A_g}$$

where $F_{c\text{-}g}$ and the cover radiosity are

$$F_{c-g} = F_{g-c}\left(A_g/A_c\right) \approx \frac{D_c^2}{4R^2 + D_c^2} \cdot \frac{A_g}{A_c} \qquad\qquad J_c = \varepsilon_c E_b\left(T_c\right) + \rho_c G_c$$

but $G_c = E_b\left(T_{sur}\right)$ and $\rho_c = 1 - \alpha_c = 1 - \varepsilon_c, J_c = \varepsilon_c \sigma T_c^4 + (1-\varepsilon_c)\sigma T_{sur}^4 = (170.2 + 387.4)\text{ W/m}^2$. Hence, the irradiation is

$$G_g = G_{g,a} + \frac{1}{A_g}\left(\frac{D_c^2}{4R^2 + D_c^2} \cdot \frac{A_g}{A_c}\right)\left[\varepsilon_c \sigma T_c^4 + (1-\varepsilon_c)\sigma T_{sur}^4\right] A_c$$

$$G_g = 354.4 \text{ mW/m}^2 + \left(\frac{0.10^2}{4\times 1^2 + 0.10^2}\right)\left[0.2\times\sigma(350)^4 + (1-0.2)\times\sigma(300)^4\right] \text{W/m}^2$$

$$G_g = 354.4 \text{ mW/m}^2 + 424.4 \text{ mW/m}^2 + 916.2 \text{ mW/m}^2 = 1,695 \text{ mW/m}^2.$$

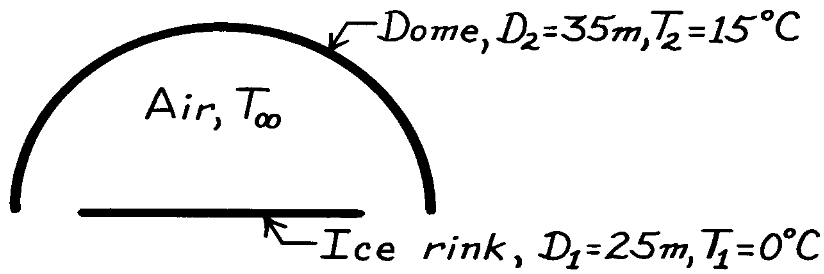
COMMENTS: (1) Note we have assumed $A_f << A_c$ so that effect of the aperture is negligible. (2) In part (b), the irradiation due to radiosity from the shield can be written also as $G_{g,c} = q_{c\text{-}g}/A_g = (J_c/\pi)\cdot A_c \cdot \omega_{g\text{-}c}/A_g$ where $\omega_{g\text{-}c} = A_g/R^2$. This is an excellent approximation since $A_c << R^2$.

PROBLEM 13.17

KNOWN: Temperature and diameters of a circular ice rink and a hemispherical dome.

FIND: Net rate of heat transfer to the ice due to radiation exchange with the dome.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody behavior for dome and ice.

ANALYSIS: From Eq. 13.13 the net rate of energy exchange between the two blackbodies is

$$q_{21} = A_2 F_{21} \sigma \left(T_2^4 - T_1^4\right)$$

From reciprocity, $A_2\,F_{21} = A_1\,F_{12} = \left(\pi D_1^2 / 4\right)1$

$$A_2 F_{21} = (\pi/4)(25\text{ m})^2 1 = 491\text{ m}^2.$$

Hence

$$q_{21} = 491\text{ m}^2 \left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left[(288\text{ K})^4 - (273\text{ K})^4\right]$$

$$q_{21} = 3.69\times10^4\text{ W.}$$ <

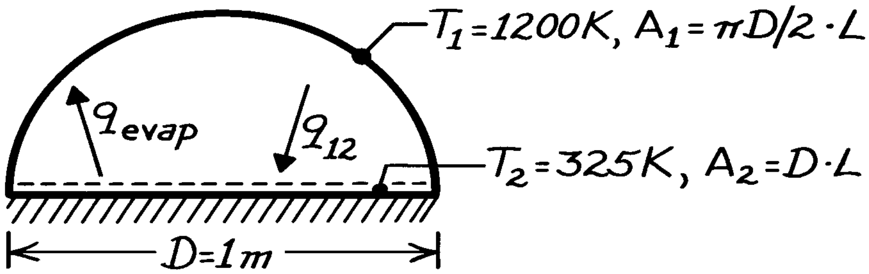
COMMENTS: If the air temperature, T_∞, exceeds T_1, there will also be heat transfer by convection to the ice. The radiation and convection transfer to the ice determine the heat load which must be handled by the cooling system.

PROBLEM 13.18

KNOWN: Surface temperature of a semi-circular drying oven.

FIND: Drying rate per unit length of oven.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody behavior for furnace wall and water, (2) Convection effects are negligible and bottom is insulated.

PROPERTIES: *Table A-6*, Water (325 K): $h_{fg} = 2.378 \times 10^6 \text{ J/kg}$.

ANALYSIS: Applying a surface energy balance,

$$q_{12} = q_{evap} = \dot{m} h_{fg}$$

where it is assumed that the net radiation heat transfer to the water is balanced by the evaporative heat loss. From Eq. 13.13

$$q_{12} = A_1 F_{12} \sigma \left(T_1^4 - T_2^4\right).$$

From inspection and the reciprocity relation, Eq. 13.3,

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{D \cdot L}{(\pi D/2) \cdot L} \times 1 = 0.637.$$

Hence

$$\dot{m}' = \frac{\dot{m}}{L} = \frac{\pi D}{2} F_{12} \sigma \frac{\left(T_1^4 - T_2^4\right)}{h_{fg}}$$

$$\dot{m}' = \frac{\pi (1 \text{ m})}{2} \times 0.637 \times 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} \frac{(1200 \text{ K}) - (325 \text{ K})^4}{2.378 \times 10^6 \text{ J/kg}}$$

or

$$\dot{m}' = 0.0492 \text{ kg/s} \cdot \text{m}. \qquad <$$

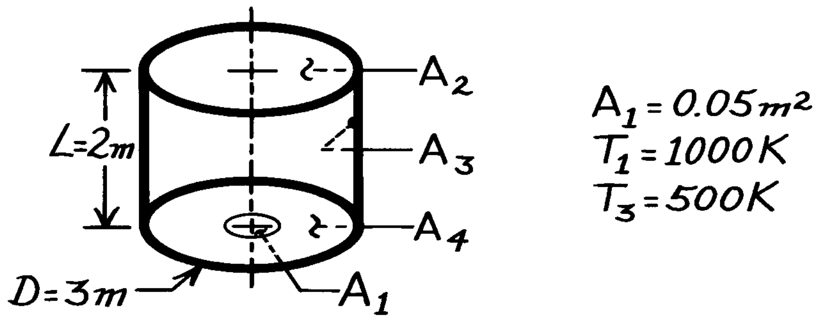
COMMENTS: Air flow through the oven is needed to remove the water vapor. The water surface temperature, T_2, is determined by a balance between radiation heat transfer to the water and the convection of latent and sensible energy from the water.

PROBLEM 13.19

KNOWN: Arrangement of three black surfaces with prescribed geometries and surface temperatures.

FIND: (a) View factor F_{13}, (b) Net radiation heat transfer from A_1 to A_3.

SCHEMATIC:



ASSUMPTIONS: (1) Interior surfaces behave as blackbodies, (2) $A_2 >> A_1$.

ANALYSIS: (a) Define the enclosure as the interior of the cylindrical form and identify A_4. Applying the view factor summation rule, Eq. 13.4,

$$F_{11} + F_{12} + F_{13} + F_{14} = 1. \quad (1)$$

Note that $F_{11} = 0$ and $F_{14} = 0$. From Eq. 13.8,

$$F_{12} = \frac{D^2}{D^2 + 4L^2} = \frac{(3\text{m})^2}{(3\text{m})^2 + 4(2\text{m})^2} = 0.36. \quad (2)$$

From Eqs. (1) and (2),

$$F_{13} = 1 - F_{12} = 1 - 0.36 = 0.64.$$ <

(b) The net heat transfer rate from A_1 to A_3 follows from Eq. 13.13,

$$q_{13} = A_1\ F_{13}\ \sigma\left(T_1^4 - T_3^4\right)$$

$$q_{13} = 0.05\text{m}^2 \times 0.64 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(1000^4 - 500^4\right)\text{K}^4 = 1700\,\text{W}.$$ <

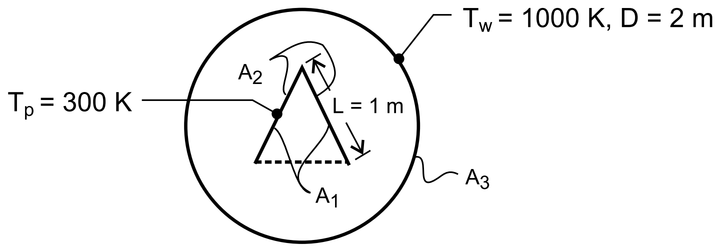
COMMENTS: Note that the summation rule, Eq. 13.4, applies to an enclosure; that is, the total region above the surface must be considered.

PROBLEM 13.20

KNOWN: Furnace diameter and temperature. Dimensions and temperature of suspended part.

FIND: Net rate of radiation transfer per unit length to the part.

SCHEMATIC:



ASSUMPTIONS: (1) All surfaces may be approximated as blackbodies.

ANALYSIS: From symmetry considerations, it is convenient to treat the system as a three-surface enclosure consisting of the inner surfaces of the vee (1), the outer surfaces of the vee (2) and the furnace wall (3). The net rate of radiation heat transfer to the part is then

$$q'_{1,2} = A'_3 F_{31} \sigma \left(T_w^4 - T_p^4\right) + A'_3 F_{32} \sigma \left(T_w^4 - T_p^4\right)$$

From reciprocity,

$$A'_3 F_{31} = A'_1 F_{13} = 2L \times 0.5 = 1\text{m}$$

where surface 3 may be represented by the dashed line and, from symmetry, $F_{13} = 0.5$. Also,

$$A'_3 F_{32} = A'_2 F_{23} = 2L \times 1 = 2\text{m}$$

Hence,

$$q'_{1,2} = (1+2)\text{m} \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left(1000^4 - 300^4\right)\text{K}^4 = 1.69 \times 10^5\ \text{W/m} \qquad <$$

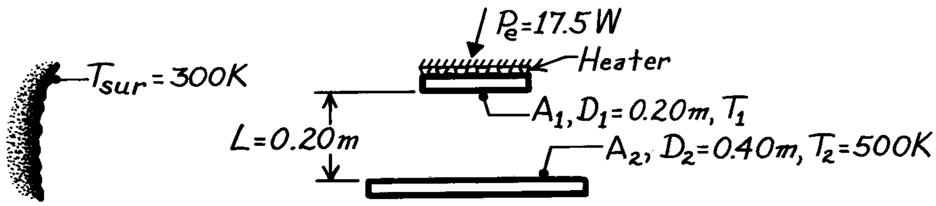
COMMENTS: With all surfaces approximated as blackbodies, the result is independent of the tube diameter. Note that $F_{11} = 0.5$.

PROBLEM 13.21

KNOWN: Coaxial, parallel black plates with surroundings. Lower plate (A_2) maintained at prescribed temperature T_2 while electrical power is supplied to upper plate (A_1).

FIND: Temperature of the upper plate T_1.

SCHEMATIC:



ASSUMPTIONS: (1) Plates are black surfaces of uniform temperature, and (2) Backside of heater on A_1 insulated.

ANALYSIS: The net radiation heat rate leaving A_i is

$$P_e = \sum_{j=1}^{N} q_{ij} = A_1 F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1 F_{13}\sigma\left(T_1^4 - T_3^4\right)$$

$$P_e = A_1\sigma\left[F_{12}\left(T_1^4 - T_2^4\right) + F_{13}\left(T_1^4 - T_{sur}^4\right)\right] \qquad (1)$$

From Fig. 13.5 for coaxial disks (see Table 13.2),

$$R_1 = r_1/L = 0.10\ \text{m}/0.20\ \text{m} = 0.5 \qquad R_2 = r_2/L = 0.20\ \text{m}/0.20\ \text{m} = 1.0$$

$$S = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+1^2}{(0.5)^2} = 9.0$$

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(r_2/r_1\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{9 - \left[9^2 - 4\left(0.2/0.1\right)^2\right]^{1/2}\right\} = 0.469.$$

From the summation rule for the enclosure A_1, A_2 and A_3 where the last area represents the surroundings with $T_3 = T_{sur}$,

$$F_{12} + F_{13} = 1 \qquad F_{13} = 1 - F_{12} = 1 - 0.469 = 0.531.$$

Substituting numerical values into Eq. (1), with $A_1 = \pi D_1^2/4 = 3.142\times10^{-2}\,\text{m}^2$,

$$17.5\ \text{W} = 3.142\times10^{-2}\,\text{m}^2 \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[0.469\left(T_1^4 - 500^4\right)\text{K}^4 + 0.531\left(T_1^4 - 300^4\right)\text{K}^4\right]$$

$$9.823\times10^9 = 0.469\left(T_1^4 - 500^4\right) + 0.531\left(T_1^4 - 300^4\right)$$

find by trial-and-error that $T_1 = 456\ \text{K}.$ $<$

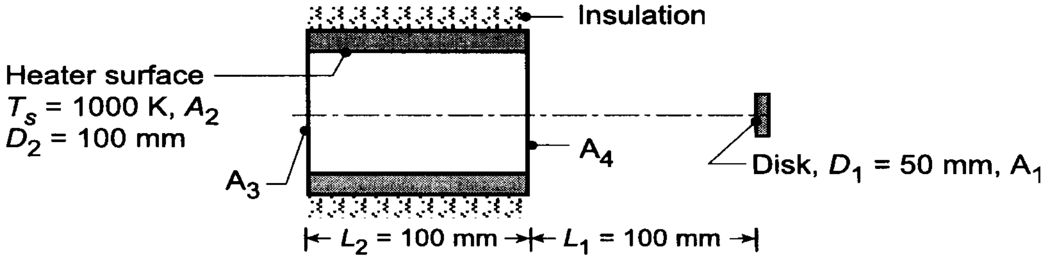
COMMENTS: Note that if the upper plate were adiabatic, $T_1 = 427$ K.

PROBLEM 13.22

KNOWN: Tubular heater radiates like blackbody at 1000 K.

FIND: (a) Radiant power from the heater surface, A_s, intercepted by a disc, A_1, at a prescribed location $q_{s\to1}$; irradiation on the disk, G_1; and (b) Compute and plot $q_{s\to1}$ and G_1 as a function of the separation distance L_1 for the range $0 \le L_1 \le 200$ mm for disk diameters D_1 = 25, and 50 and 100 mm.

SCHEMATIC:



ASSUMPTIONS: (1) Heater surface behaves as blackbody with uniform temperature.

ANALYSIS: (a) The radiant power leaving the inner surface of the tubular heater that is intercepted by the disk is

$$q_{2\to1} = (A_2 E_{b2}) F_{21} \qquad (1)$$

where the heater is surface 2 and the disk is surface 1. It follows from the reciprocity rule, Eq. 13.3, that

$$F_{21} = \frac{A_1}{A_2} F_{12}. \qquad (2)$$

Define now the hypothetical disks, A_3 and A_4, located at the ends of the tubular heater. By inspection, it follows that

$$F_{14} = F_{12} + F_{13} \qquad \text{or} \qquad F_{12} = F_{14} - F_{13} \qquad (3)$$

where F_{14} and F_{13} may be determined from Fig. 13.5. Substituting numerical values, with $D_3 = D_4 = D_2$,

$$F_{13} = 0.08 \qquad \text{with} \qquad \frac{L}{r_i} = \frac{L_1 + L_2}{D_1/2} = \frac{200}{50/2} = 8 \qquad \frac{r_j}{L} = \frac{D_3/2}{L_1 + L_2} = \frac{100/2}{200} = 0.25$$

$$F_{14} = 0.20 \qquad \text{with} \qquad \frac{L}{r_i} = \frac{L_1}{D_1/2} = \frac{100}{50/2} = 4 \qquad \frac{r_j}{L} = \frac{D_4/2}{L_1} = \frac{100/2}{100} = 0.5$$

Substituting Eq. (3) into Eq. (2) and then into Eq. (1), the result is

$$q_{2\to1} = A_1 (F_{14} - F_{13}) E_{b2}$$

$$q_{2\to1} = \left[\pi \left(50\times10^{-3}\right)^2 \text{m}^2/4\right](0.20 - 0.08)\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 (1000\ \text{K})^4 = 13.4\text{W} \qquad <$$

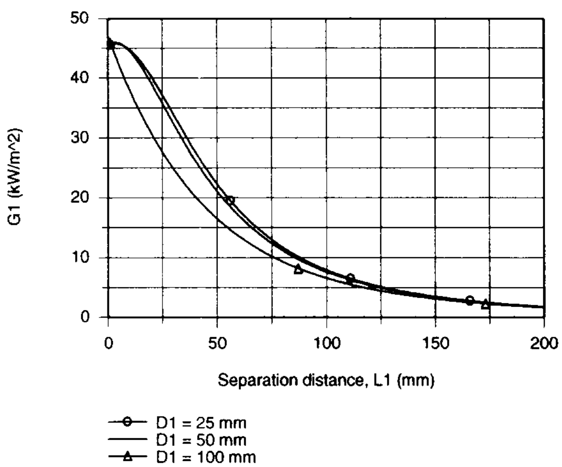
where $E_{b2} = \sigma T_s^4$. The irradiation G_1 originating from emission leaving the heater surface is

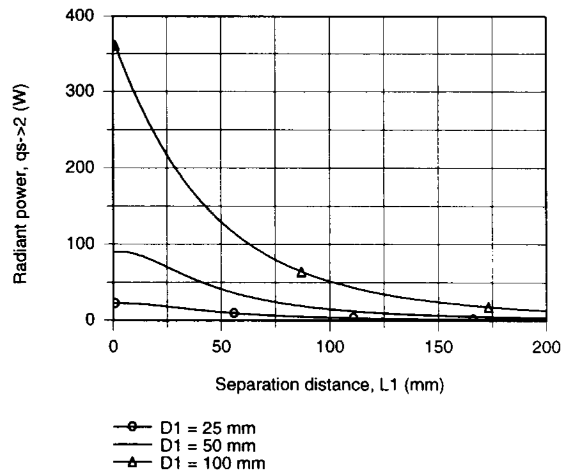
$$G_1 = \frac{q_{s\to1}}{A_1} = \frac{13.4\ \text{W}}{\pi(0.050\ \text{m})^2/4} = 6825\,\text{W/m}^2. \qquad (4) <$$

Continued …..

PROBLEM 13.22 (Cont.)

(b) Using the foregoing equations in *IHT* along with the *Radiation Tool-View Factors* for *Coaxial Parallel Disks*, G_1 and $q_{s\to1}$ were computed as a function of L_1 for selected values of D_1. The results are plotted below.





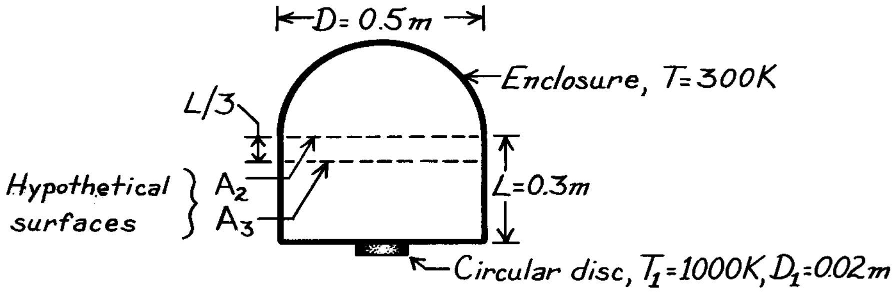
In the upper left-hand plot, G_1 decreases with increasing separation distance. For a given separation distance, the irradiation decreases with increasing diameter. With values of D_1 = 25 and 50 mm, the irradiation values are only slightly different, which diminishes as L_1 increases. In the upper right-hand plot, the radiant power from the heater surface reaching the disk, $q_{s\to2}$, decreases with increasing L_1 and decreasing D_1. Note that while G_1 is nearly the same for D_1 = 25 and 50 mm, their respective $q_{s\to2}$ values are quite different. Why is this so?

PROBLEM 13.23

KNOWN: Dimensions and temperatures of an enclosure and a circular disc at its base.

FIND: Net radiation heat transfer between the disc and portions of the enclosure.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody behavior for disc and enclosure surfaces, (2) Area of disc is much less than that of the hypothetical surfaces, $(A_1/A_2) << 1$ and $(A_1/A_3) << 1$.

ANALYSIS: From Eq. 13.13 the net radiation exchange between the disc (1) and the hemispherical dome (d) is

$$q_{1d} = A_1 F_{1d}\,\sigma\left(T_1^4 - T^4\right).$$

However, since all of the radiation intercepted by the dome must pass through the hypothetical area A_2, it follows from Eq. 13.8 of Example 13.1,

$$F_{1d} = F_{12} \approx \frac{D^2}{4L^2 + D^2} = \frac{1}{(2L/D)^2 + 1} = \frac{1}{1.44 + 1} = 0.410.$$

Hence

$$q_{1d} = \frac{\pi}{4}(0.02\text{ m})^2 \times 0.41 \times 5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}\left[(1000\text{ K})^4 - (300\text{ K})^4\right]$$

$$q_{1d} = 7.24\text{ W}. \qquad <$$

Similarly, the net radiation exchange between the disc (1) and the cylindrical ring (r) of length L/3 is

$$q_{1r} = A_1\,F_{1r}\,\sigma\left(T_1^4 - T^4\right)$$

where

$$F_{1r} = F_{13} - F_{12} = \frac{D^2}{4(2L/3)^2 + D^2} - 0.41 = 0.61 - 0.41 = 0.20.$$

Hence

$$q_{1r} = \frac{\pi}{4}(0.02\text{ m})^2 \times 0.2 \times 5.67 \times 10^{-8}\text{W/m}^2\cdot\text{K}^4\left[(1000\text{ K})^4 - (300\text{ K})^4\right]$$

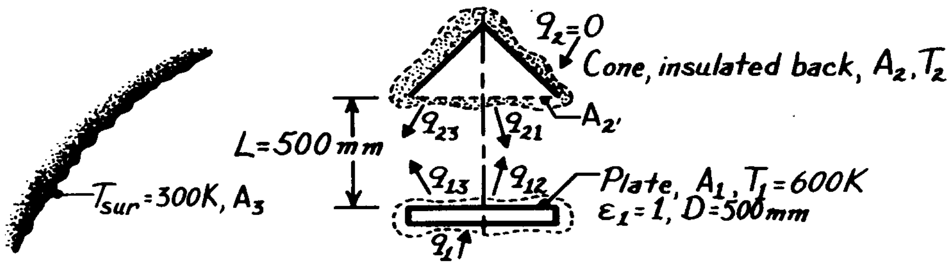
$$q_{1r} = 3.53\text{ W}. \qquad <$$

PROBLEM 13.24

KNOWN: Circular plate (A_1) maintained at 600 K positioned coaxially with a conical shape (A_2) whose backside is insulated. Plate and cone are black surfaces and located in large, insulated enclosure at 300 K.

FIND: (a) Temperature of the conical surface T_2 and (b) Electric power required to maintain plate at 600 K.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Plate and cone are black, (3) Cone behaves as insulated, reradiating surface, (4) Surroundings are large compared to plate and cone.

ANALYSIS: (a) Recognizing that the plate, cone, and surroundings from a three-(black) surface enclosure, perform a radiation balance on the cone.

$$q_2 = 0 = q_{23} + q_{21} = A_2 F_{23} \sigma \left(T_2^4 - T_3^4\right) + A_2 F_{21} \sigma \left(T_2^4 - T_1^4\right)$$

where the view factor F_{21} can be determined from the *coaxial parallel disks* relation (Table 13.2 or Fig. 13.5) with $R_i = r_i/L=250/500 = 0.5$, $R_j = 0.5$, $S = 1 + \left(1+R_j^2\right)/R_i^2 = 1 + (1 + 0.5^2)/0.5^2 = 6.00$, and noting $F_{2'1} = F_{21}$,

$$F_{21} = 0.5\left\{S - \left[S^2 - 4\left(r_j/r_i\right)^2\right]^{1/2}\right\} = 0.5\left\{6 - \left[6^2 - 4\left(0.5/0.5\right)^2\right]^{1/2}\right\} = 0.172.$$

For the enclosure, the summation rule provides, $F_{2'3} = 1 - F_{2'1} = 1 - 0.172 = 0.828$. Hence,

$$0.828\left(T_2^4 - 300^4\right) = 0 + 0.172\left(T_2^4 - 600^4\right)$$

$$T_2 = 413 \text{ K.}$$ <

(b) The power required to maintain the plate at T_2 follows from a radiation balance,

$$q_1 = q_{12} + q_{13} = A_1 F_{12} \sigma \left(T_1^4 - T_2^4\right) + A_1 F_{13} \sigma \left(T_1^4 - T_3^4\right)$$

where $F_{12} = A_{2'} F_{2'1} / A_1 = F_{21} = 0.172$ and $F_{13} = 1 - F_{12} = 0.828$,

$$q_1 = \left(\pi 0.5^2/4\right)\text{m}^2 \sigma \left[0.172\left(600^4 - 413^4\right)\text{K}^4 + 0.828\left(600^4 - 300^4\right)\text{K}^4\right]$$

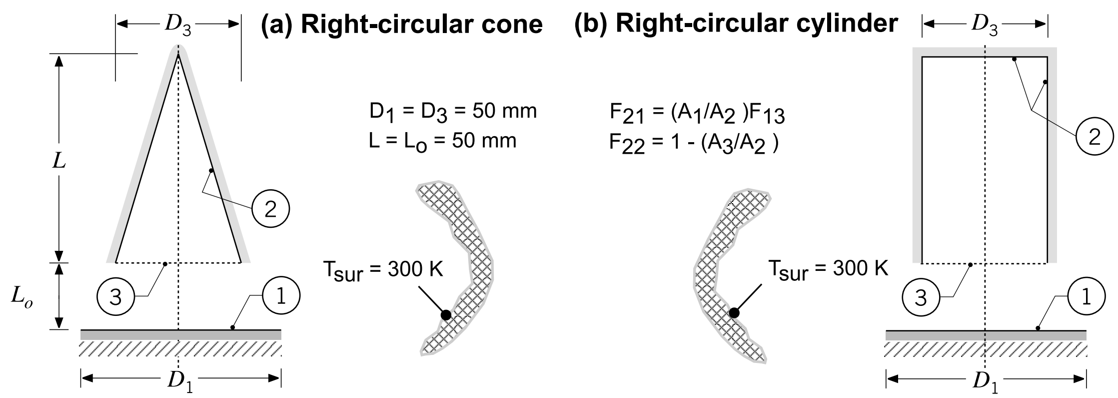
$$q_1 = 1312 \text{ W.}$$ <

PROBLEM 13.25

KNOWN: Conical and cylindrical furnaces (A_2) as illustrated and dimensioned in Problem 13.2 (S) supplied with power of 50 W. Workpiece (A_1) with insulated backside located in large room at 300 K.

FIND: Temperature of the workpiece, T_1, and the temperature of the inner surfaces of the furnaces, T_2. Use expressions for the view factors F_{21} and F_{22} given in the statement for Problem 13.2 (S).

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse, black surfaces with uniform radiosities, (2) Backside of workpiece is perfectly insulated, (3) Inner base and lateral surfaces of the cylindrical furnace treated as single surface, (4) Negligible convection heat transfer, (5) Room behaves as large, isothermal surroundings.

ANALYSIS: Considering the furnace surface (A_2), the workpiece (A_1) and the surroundings (A_s) as an enclosure, the net radiation transfer from A_1 and A_2 follows from Eq. 13.14,

Workpiece $$q_1 = 0 = A_1 F_{12}\left(E_{b1} - E_{b2}\right) + A_1 F_{1s}\left(E_{b1} - E_{bs}\right) \tag{1}$$

Furnace $$q_2 = 50\ \text{W} = A_2 F_{21}\left(E_{b2} - E_{b1}\right) + A_2 F_{2s}\left(E_{b2} - E_{bs}\right) \tag{2}$$

where $E_b = \sigma T^4$ and $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$. From summation rules on A_1 and A_2, the view factors F_{1s} and F_{2s} can be evaluated. Using reciprocity, F_{12} can be evaluated.

$$F_{1s} = 1 - F_{12} \qquad F_{2s} = 1 - F_{21} - F_{22} \qquad F_{12} = \left(A_2 / A_1\right) F_{21}$$

The expressions for F_{21} and F_{22} are provided in the schematic. With $A_1 = \pi D_1^2 / 4$ the A_2 are:

Cone: $$A_2 = \pi D_3 / 2\left(L^2 + \left(D_3 / 2\right)^2\right)^{1/2}$$ *Cylinder*: $$A_2 = \pi D_3^2 / 4 + \pi D_3 L$$

Examine Eqs (1) and (2) and recognize that there are two unknowns, T_1 and T_2, and the equations must be solved simultaneously. Using the foregoing equations in the *IHT* workspace, the results are

$$T_1 = 544\ \text{K} \qquad T_2 = 828\ \text{K}$$ <

COMMENTS: (1) From the *IHT* analysis, the relevant view factors are: F_{12} = 0.1716; F_{1s} = 0.8284; *Cone:* F_{21} = 0.07673, F_{22} = 0.5528; *Cylinder:* F_{21} = 0.03431, F_{22} =0.80.

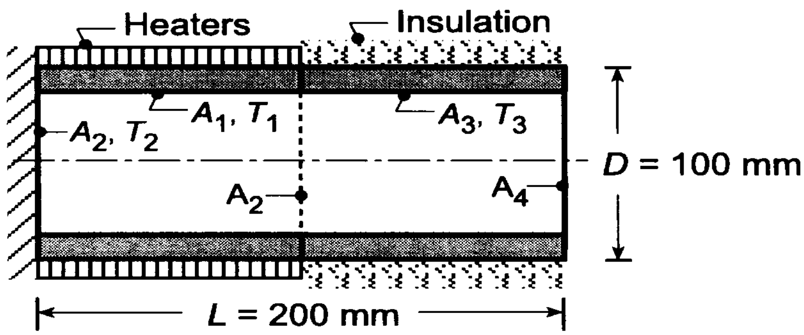
(2) That both furnace configurations provided identical results may not, at first, be intuitively obvious. Since both furnaces (A_2) are black, they can be represented by the hypothetical black area A_3 (the opening of the furnaces). As such, the analysis is for an enclosure with the workpiece (A_1), the furnace represented by the disk A_3 (at T_2), and the surroundings. As an exercise, perform this analysis to confirm the above results.

PROBLEM 13.26

KNOWN: Furnace constructed in three sections: insulated circular (2) and cylindrical (3) sections, as well as, an intermediate cylindrical section (1) with imbedded electrical resistance heaters. Cylindrical sections (1,3) are of equal length.

FIND: (a) Electrical power required to maintain the heated section at T_1 = 1000 K if all the surfaces are black, (b) Temperatures of the insulated sections, T_2 and T_3, and (c) Compute and plot q_1, T_2 and T_3 as functions of the length-to-diameter ratio, with $1 \le L/D \le 5$ and D = 100 mm.

SCHEMATIC:



ASSUMPTIONS: (1) All surfaces are black, (2) Areas (1, 2, 3) are isothermal.

ANALYSIS: (a) To complete the enclosure representing the furnace, define the hypothetical surface A_4 as the opening at 0 K with unity emissivity. For each of the enclosure surfaces 1, 2, and 3, the energy balances following Eq. 13.13 are

$$q_1 = A_1F_{12}\left(E_{b1} - E_{b2}\right) + A_1F_{13}\left(E_{b1} - E_{b3}\right) + A_1F_{14} + \left(E_{b1} - E_{b4}\right) \quad (1)$$

$$0 = A_2F_{21}\left(E_{b2} - E_{b1}\right) + A_2F_{23}\left(E_{b2} - E_{b3}\right) + A_2F_{24}\left(E_{b2} - E_{b4}\right) \quad (2)$$

$$0 = A_3F_{31}\left(E_{b3} - E_{b1}\right) + A_3F_{32}\left(E_{b3} - E_{b2}\right) + A_3F_{34}\left(E_{b3} - E_{b4}\right) \quad (3)$$

where the emissive powers are

$$E_{b1} = \sigma T_1^4 \qquad E_{b2} = \sigma T_2^4 \qquad E_{b3} = \sigma T_3^4 \qquad E_{b4} = 0 \quad (4-7)$$

For this four surface enclosure, there are N^2 = 16 view factors and N (N – 1)/2 = 4 × 3/2 = 6 must be directly determined (by inspection or formulas) and the remainder can be evaluated from the summation rule and reciprocity relation. By inspection,

$$F_{22} = 0 \qquad F_{44} = 0 \quad (8,9)$$

From the coaxial parallel disk relation, Table 13.2, find F_{24}

$$S = 1 + \frac{1 + R_4^2}{R_2^2} = 1 + \frac{1 + (0.250)^2}{(0.250)^2} = 18.00$$

$$R_2 = r_2 / L = 0.050\,m / 0.200\,m = 0.250 \qquad R_4 = r_4 / L = 0.250$$

$$F_{24} = 0.5\left\{S - \left[S^2 - 4\left(r_4 / r_2\right)^2\right]^{1/2}\right\}$$

$$F_{24} = 0.5\left\{18.00 - \left[18.00^2 - 4(1)^2\right]^{1/2}\right\} = 0.0557 \quad (10)$$

Consider the three-surface enclosure $1 - 2 - 2'$ and find F_{11} as beginning with the summation rule,

Continued …..

$$F_{11} = 1 - F_{12} - F_{12'} \tag{11}$$

where, from symmetry, $F_{12} = F_{12'}$, and using reciprocity,

$$F_{12} = A_2 F_{21} / A_1 = \left(\pi D^2 / 4\right) F_{23} / \left(\pi DL / 2\right) = DF_{21} / 2L \tag{12}$$

and from the summation rule on A_2

$$F_{21} = 1 - F_{22'} = 1 - 0.172 = 0.828\,, \tag{13}$$

Using the coaxial parallel disk relation, Table 13.2, to find F_{221},

$$S = 1 + \frac{1 + R_{2'}^2}{R_2^2} = 1 + \frac{1 + 0.50^2}{0.50^2} = 6.000$$

$$R_2 = r_2 / L = 0.050\,\text{m} / \left(0.200 / 2\,\text{m}\right) = 0.500 \qquad\qquad R_{2'} = 0.500$$

$$F_{22'} = 0.5\left\{S - \left[S^2 - 4\left(r_{2'} / r_2\right)^2\right]^{1/2}\right\}$$

$$F_{22'} = 0.5\left\{6 - \left[6^2 - 4\left(1\right)^2\right]^{1/2}\right\} = 0.1716$$

Evaluating F_{12} from Eq. (12), find

$$F_{12} = 0.100\ \text{m} \times 0.828 / 2 \times 0.200\ \text{m} = 0.2071$$

and evaluating F_{11} from Eq. (11), find

$$F_{11} = 1 - 2 \times F_{12} = 1 - 2 \times 0.207 = 0.586$$

From symmetry, recognize that $F_{33} = F_{11}$ and $F_{43} = F_{21}$. To this point we have directly determined six view factors (underlined in the matrix below) and the remaining F_{ij} can be evaluated from the summation rules and appropriate reciprocity relations. The view factors written in matrix form, $[F_{ij}]$ are.

<u>0.5858</u>	<u>0.2071</u>	0.1781	0.02896
<u>0.8284</u>	<u>0</u>	0.1158	<u>0.05573</u>
0.1781	0.02896	0.5858	0.2071
0.1158	0.05573	0.8284	<u>0</u>

Knowing all the required view factors, the energy balances and the emissive powers, Eqs. (4-6), can be solved simultaneously to obtain:

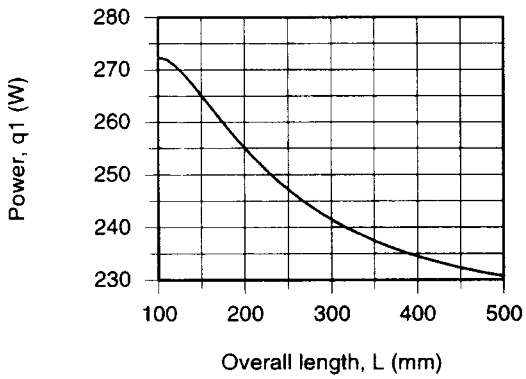
$$q_1 = 255\,\text{W} \qquad E_{b2} = 5.02 \times 10^4\,\text{W/m}^2 \qquad E_{b3} = 2.79 \times 10^4\,\text{W/m}^2 \qquad <$$

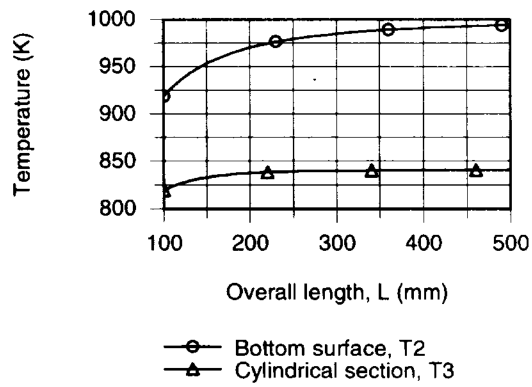
$$T_2 = 970\,\text{K} \qquad T_3 = 837.5\,\text{K} \qquad <$$

Continued …..

PROBLEM 13.26 (Cont.)

(b) Using the energy balances, Eqs. (1-3), along with the *IHT Radiation Tool, View Factors, Coaxial parallel disks*, a model was developed to calculate q_1, T_2, and T_3 as a function of length L for fixed diameter D = 100 m. The results are plotted below.





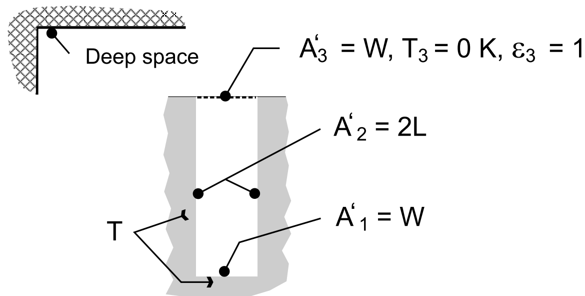
For fixed diameter, as the overall length increases, the power required to maintain the heated section at T_1 = 1000 K decreases. This follows since the furnace opening area is a smaller fraction of the enclosure surface area as L increases. As L increases, the bottom surface temperature T_2 increases as L increases and, in the limit, will approach that of the heated section, T_1 = 1000 K. As L increases, the temperature of the insulated cylindrical section, T_3, increases, but only slightly. The limiting value occurs when $E_{b3} = 0.5 \times E_{b1}$ for which $T_3 \to 840$ K. Why is that so?

PROBLEM 13.27

KNOWN: Dimensions and temperature of a rectangular fin array radiating to deep space.

FIND: Expression for rate of radiation transfer per unit length from a unit section of the array.

SCHEMATIC:



ASSUMPTIONS: (1) Surfaces may be approximated as blackbodies, (2) Surfaces are isothermal, (3) Length of array (normal to page) is much larger than W and L.

ANALYSIS: Deep space may be represented by the hypothetical surface A_3', which acts as a blackbody at absolute zero temperature. The net rate of radiation heat transfer to this surface is therefore equivalent to the rate of heat rejection by a unit section of the array.

$$q_3' = A_1' F_{13} \sigma \left(T_1^4 - T_3^4\right) + A_2' F_{23} \sigma \left(T_2^4 - T_3^4\right)$$

With $A_2' F_{23} = A_3' F_{32} = A_1' F_{12}$, $T_1 = T_2 = T$ and $T_3 = 0$,

$$q_3' = A_1' \left(F_{13} + F_{12}\right) \sigma T^4 = W \sigma T^4$$ <

Radiation from a unit section of the array corresponds to emission from the base. Hence, if blackbody behavior can, indeed, be maintained, the fins do nothing to enhance heat rejection.

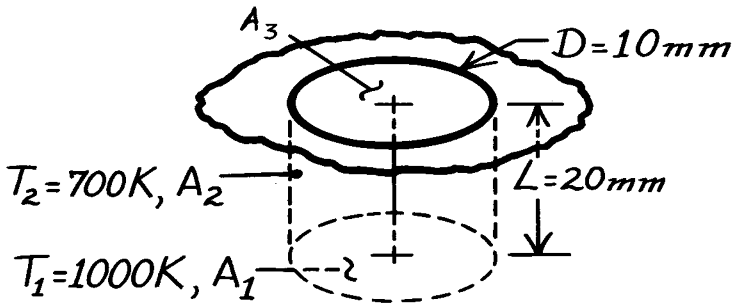
COMMENTS: (1) The foregoing result should come as no surprise since the surfaces of the unit section form an isothermal blackbody cavity for which emission is proportional to the area of the opening. (2) Because surfaces 1 and 2 have the same temperature, the problem could be treated as a two-surface enclosure consisting of the combined (1, 2) and 3. It follows that $q_3' = q_{(1,2)3}' = A_{(1,2)}' F_{(1,2)3} \sigma T^4 = A_3' F_{3(1,2)} \sigma T^4 = W \sigma T^4$, (3) If blackbody behavior cannot be achieved $(\varepsilon_1, \varepsilon_2 < 1)$, enhancement would be afforded by the fins.

PROBLEM 13.28

KNOWN: Dimensions and temperatures of side and bottom walls in a cylindrical cavity.

FIND: Emissive power of the cavity.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody behavior for surfaces 1 and 2

ANALYSIS: The desired emissive power is defined as

$$E = q_3 / A_3$$

where

$$q_3 = A_1 F_{13} E_{b1} + A_2 F_{23} E_{b2}.$$

From symmetry, $F_{23} = F_{21}$, and from reciprocity, $F_{21} = (A_1/A_2) F_{12}$. With $F_{12} = 1 - F_{13}$, it follows that

$$q_3 = A_1 F_{13} E_{b1} + A_1 (1 - F_{13}) E_{b2} = A_1 E_{b2} + A_1 F_{13} (E_{b1} - E_{b2}).$$

Hence, with $A_1 = A_3$,

$$E = \frac{q_3}{A_3} = E_{b2} + F_{13}(E_{b1} - E_{b2}) = \sigma T_2^4 + F_{13} \sigma \left(T_1^4 - T_2^4\right).$$

From Fig. 13.15, with $(L/r_i) = 4$ and $(r_j/L) = 0.25$, $F_{13} \approx 0.05$. Hence

$$E = 5.67 \times 10^{-8} W/m^2 \cdot K^4 + 0.05 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(1000^4 - 700^4\right) K^4$$

$$E = 1.36 \times 10^4 W/m^2 + 0.22 \times 10^4 W/m^2$$

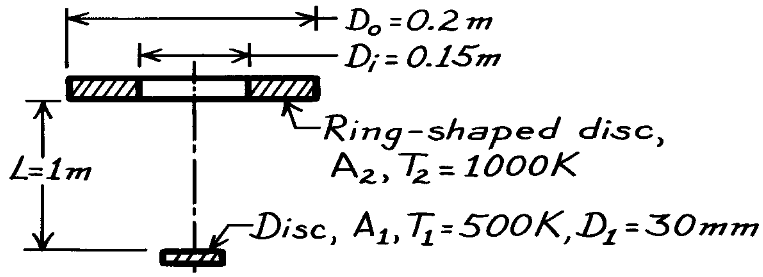
$$E = 1.58 \times 10^4 W/m^2.$$ <

PROBLEM 13.29

KNOWN: Aligned, parallel discs with prescribed geometry and orientation.

FIND: Net radiative heat exchange between the discs.

SCHEMATIC:



ASSUMPTIONS: (1) Surfaces behave as blackbodies, (2) $A_1 << A_2$.

ANALYSIS: The net radiation exchange between the two black surfaces follows from Eq. 13.13 written as

$$q_{12} = A_1\ F_{12}\ \sigma\left(T_1^4 - T_2^4\right).$$

The view factor can be determined from Eq. 13.8 which is appropriate for a small disc, aligned and parallel to a much larger disc.

$$F_{ij} = \frac{D_j^2}{D_j^2 + 4L^2}$$

where D_j is the diameter of the larger disk and L is the distance of separation. It follows that

$$F_{12} = F_{1o} - F_{1i} = 0.00990 - 0.00559 = 0.00431$$

where

$$F_{1o} = D_o^4 / \left(D_o^2 + 4L^2\right) = 0.2^2\,m^2 / \left(0.2^2\,m^2 + 4\times 1\ m^2\right) = 0.00990$$

$$F_{1i} = D_i / \left(D_i^2 + 4L^2\right) = 0.15^2\,m^2 / \left(0.15^2\,m^2 + 4\times 1\ m^2\right) = 0.00559.$$

The net radiation exchange is then

$$q_{12} = \frac{\pi\,(0.03m)^2}{4} \times 0.00431 \times 5.67\times 10^{-8}\ \frac{W}{m^2\cdot K^4}\left(500^4 - 1000^4\right)K^4 = -0.162\,W.$$

COMMENTS: F_{12} can be approximated using solid angle concepts if $D_o << L$. That is, the view factor for A_1 to A_o (whose diameter is D_o) is

$$F_{1o} \approx \frac{\omega_{o-1}}{\pi} = \frac{A_o / L^2}{\pi} = \frac{\pi D_o^2}{4\pi L^2} = \frac{D_o^2}{4L^2}.$$

Numerically, $F_{1o} = 0.0100$ and it follows $F_{1i} \approx D_i^2 / 4L^2 = 0.00563$. This gives $F_{12} = 0.00437$. An analytical expression can be obtained from Ex. 13.1 by replacing the lower limit of integration by $D_i/2$, giving

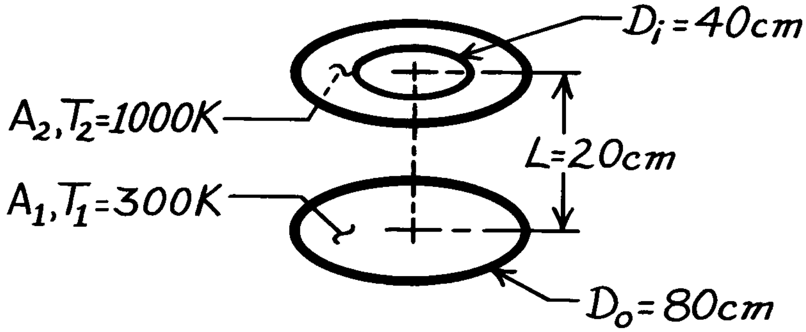
$$F_{12} = L^2\left[-1/\left(D_o^2/4 + L^2\right) + 1/\left(D_i^2/4 + L^2\right)\right] = 0.00431.$$

PROBLEM 13.30

KNOWN: Two black, plane discs, one being solid, the other ring-shaped.

FIND: Net radiative heat exchange between the two surfaces.

SCHEMATIC:



ASSUMPTIONS: (1) Discs are parallel and coaxial, (2) Discs are black, diffuse surfaces, (3) Convection effects are not being considered.

ANALYSIS: The net radiative heat exchange between the solid disc, A_1, and the ring-shaped disc, A_2, follows from Eq. 13.13.

$$q_{12} = A_1 F_{12}\, \sigma \left(T_1^4 - T_2^4\right)$$

The view factor F_{12} can be determined from Fig. 13.5 after some manipulation. Define these two hypothetical surfaces;

$$A_3 = \frac{\pi D_o^2}{4},\ \text{located co-planar with } A_2\text{, but a solid surface}$$

$$A_4 = \frac{\pi D_i^2}{4},\ \text{located co-planar with } A_2\text{, representing the missing center.}$$

From view factor relations and Fig. 13.5, it follows that

$$F_{12} = F_{13} - F_{14} = 0.62 - 0.20 = 0.42$$

F_{14}: $\quad \frac{r_j}{L} = \frac{40/2}{20} = 1, \qquad \frac{L}{r_i} = \frac{20}{80/2} = 0.5, \qquad F_{14} = 0.20$

F_{13}: $\quad \frac{r_j}{L} = \frac{80/2}{20} = 2, \qquad \frac{L}{r_i} = \frac{20}{80/2} = 0.5, \qquad F_{13} = 0.62.$

Hence

$$q_{12} = \left(\pi 0.80^2 / 4\right) m^2 \times 0.42 \times 5.67 \times 10^{-8}\, W/m^2 \cdot K^4 \left(300^4 - 1000^4\right) K^4$$

$$q_{12} = -11.87\ kW.$$ <

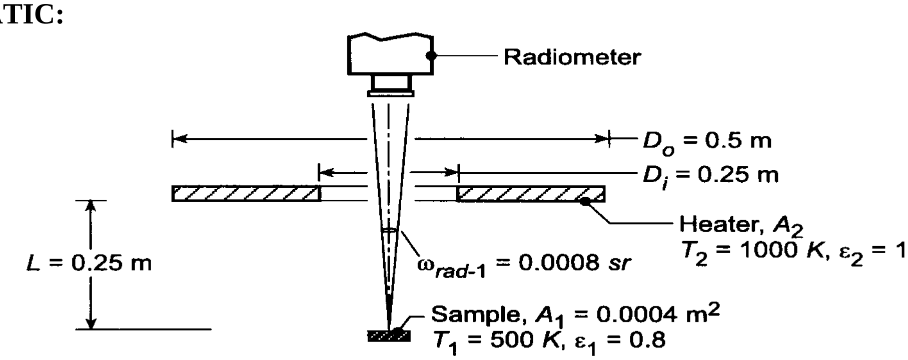
Assuming negligible radiation exchange with the surroundings, the negative sign implies that q_1 = -11.87 kW and q_2 = +11.87 kW.

PROBLEM 13.31

KNOWN: Radiometer viewing a small target area (1), A_1, with a solid angle $\omega = 0.0008$ sr. Target has an area $A_1 = 0.004\ m^2$ and is diffuse, gray with emissivity $\varepsilon = 0.8$. The target is heated by a ring-shaped disc heater (2) which is black and operates at $T_2 = 1000$ K.

FIND: (a) Expression for the radiant power leaving the target which is collected by the radiometer in terms of the target radiosity, J_1, and relevant geometric parameters; (b) Expression for the target radiosity in terms of its irradiation, emissive power and appropriate radiative properties; (c) Expression for the irradiation on the target, G_1, due to emission from the heater in terms of the heater emissive power, the heater area and an appropriate view factor; numerically evaluate G_1; and (d) Determine the radiant power collected by the radiometer using the foregoing expressions and results.

SCHEMATIC:



ASSUMPTIONS: (1) Target is diffuse, gray, (2) Target area is small compared to the square of the separation distance between the sample and the radiometer, and (3) Negligible irradiation from the surroundings onto the target area.

ANALYSIS: (a) From Eq. (12.5) with $I_1 = I_{1,e+r} = J_1/\pi$, the radiant power leaving the target collected by the radiometer is

$$q_{1\rightarrow rad} = \frac{J_1}{\pi} A_1 \cos\theta_1 \omega_{rad-1} \qquad < \quad (1)$$

where $\theta_1 = 0°$ and ω_{rad-1} is the solid angle the radiometer subtends with respect to the target area.

(b) From Eq. 13.16, the radiosity is the sum of the emissive power plus the reflected irradiation.

$$J_1 = E_1 + \rho G_1 = \varepsilon E_{b,1} + (1-\varepsilon) G_1 \qquad < \quad (2)$$

where $E_{b1} = \sigma T_1^4$ and $\rho = 1 - \varepsilon$ since the target is diffuse, gray.

(c) The irradiation onto G_1 due to emission from the heater area A_2 is

$$G_1 = \frac{q_{2\rightarrow 1}}{A_1}$$

where $q_{2\rightarrow 1}$ is the radiant power leaving A_2 which is intercepted by A_1 and can be written as

$$q_{2\rightarrow 1} = A_2 F_{21} E_{b2} \qquad (3)$$

where $E_{b2} = \sigma T_2^4$. F_{21} is the fraction of radiant power leaving A_2 which is intercepted by A_1. The view factor F_{12} can be written as

Continued …..

$$F_{12} = F_{1-o} \qquad F_{1-i} = 0.5 - 0.2 = 0.3$$

where from Eq. 13.8,

$$F_{1-o} = \frac{D_o^2}{D_o^2 + 4L^2} = \frac{0.5^2}{0.5^2 + 4(0.25)^2} = 0.5 \tag{3}$$

$$F_{1-i} = \frac{D_i^2}{D_1^2 + 4L^2} = \frac{0.25^2}{0.25^2 + 4(0.25)^2} = 0.2$$

and from the reciprocity rule,

$$F_{21} = \frac{A_1 F_{12}}{A_2} = \frac{0.0004\,m^2 \times 0.3}{\pi/4\left(0.5^2 - 0.25^2\right)m^2} = 0.000815$$

Substituting numerical values into Eq. (3), find

$$G_1 = \frac{\pi/4\left(0.5^2 - 0.25^2\right)m^2 \times 0.000815 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\,(1000\ K)^4}{0.0004\,m^2}$$

$$G_1 = 17{,}013\,W/m^2$$ <

(d) Substituting numerical values into Eq. (1), the radiant power leaving the target collected by the radiometer is

$$q_{1\to rad} = \left(6238\,W/m^2/\pi\,sr\right) \times 0.0004\,m^2 \times 1 \times 0.0008\,sr = 635\,\mu W$$ <

where the radiosity, J_1, is evaluated using Eq. (2) and G_1.

$$J_1 = 0.8 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4 \times (500\ K)^4 + (1 - 0.8) \times 17{,}013\,W/m^2$$

$$J_1 = (2835 + 3403)\,W/m^2 = 6238\,W/m^2$$ <

COMMENTS: (1) Note that the emitted and reflected irradiation components of the radiosity, J_1, are of the same magnitude.

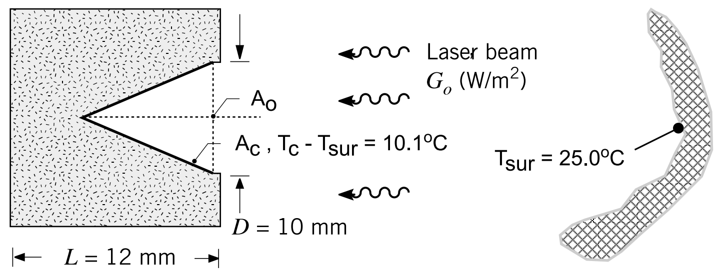
(2) Suppose the surroundings were at room temperature, T_{sur} = 300 K. Would the reflected irradiation due to the surroundings contribute significantly to the radiant power collected by the radiometer? Justify your conclusion.

PROBLEM 13.32

KNOWN: Thin-walled, black conical cavity with opening D = 10 mm and depth of L = 12 mm that is well insulated from its surroundings. Temperature of meter housing and surroundings is 25.0°C.

FIND: Optical (radiant) flux of laser beam, G_o (W/m^2), incident on the cavity when the fine-wire thermocouple indicates a temperature rise of 10.1°C.

SCHEMATIC:



ASSUMPTIONS: (1) Cavity surface is black and perfectly insulated from its mounting material in the meter, (2) Negligible convection heat transfer from the cavity surface, and (3) Surroundings are large, isothermal.

ANALYSIS: Perform an energy balance on the walls of the cavity considering absorption of the laser irradiation, absorption from the surroundings and emission.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$A_o\, G_o + A_o\, G_{sur} - A_o\, E_b\left(T_c\right) = 0$$

where $A_o = \pi D^2/4$ represents the opening of the cavity. All of the radiation entering or leaving the cavity passes through this hypothetical surface. Hence, we can treat the cavity as a black disk at T_c. Since $G_{sur} = E_b\,(T_{sur})$, and $E_b = \sigma T^4$ with $\sigma = 5.67 \times 10^{-8}$ $W/m^2 \cdot K^4$, the energy balance has the form

$$G_o + \sigma(25.0 + 273)^4 K^4 - \sigma(25.0 + 10.1 + 273)^4 K^4 = 0$$

$$G_o = 63.8\ W/m^2$$ <

PROBLEM 13.33

KNOWN: Electrically heated sample maintained at T_s = 500 K with diffuse, spectrally selective coating. Sample is irradiated by a furnace located coaxial to the sample at a prescribed distance. Furnace has isothermal walls at T_f = 3000 K with ε_f = 0.7 and an aperture of 25 mm diameter. Sample experiences convection with ambient air at T_∞ = 300 K and h = 20 W/m^2·K. The surroundings of the sample are large with a uniform temperature T_{sur} = 300 K. A radiation detector sensitive to only power in the spectral region 3 to 5 μm is positioned at a prescribed location relative to the sample.

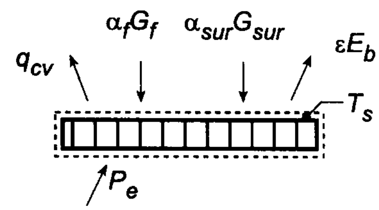
FIND: (a) Electrical power, P_e, required to maintain the sample at T_s = 500 K, and (b) Radiant power incident on the detector within the spectral region 3 to 5 μm considering both emission and reflected irradiation from the sample.

SCHEMATIC:

[redacted]

ASSUMPTIONS: (1) Steady-state condition, (2) Furnace is large, isothermal enclosure with small aperture and radiates as a blackbody, (3) Sample coating is diffuse, spectrally selective, (4) Sample and detector areas are small compared to their separation distance squared, (5) Surroundings are large, isothermal.

ANALYSIS: (a) Perform an energy balance on the sample mount, which experiences electrical power dissipation, convection with ambient air, absorbed irradiation from the furnace, absorbed irradiation from the surroundings and emission,



$$\dot{E}'_{in} - \dot{E}'_{out} = 0$$

$$P_e + \left[-h\left(T_s - T_\infty\right) + \alpha_1 G_f + \alpha_{sur} G_{sur} - \varepsilon E_b\left(T_s\right)\right] A_s = 0 \quad (1)$$

where $E_b\left(T_s\right) = \sigma T_s^4$ and $A_s = \pi D_s^2 / 4$.

Irradiations on the sample: The irradiation from the furnace aperture onto the sample can be written as

$$G_f = \frac{q_{f\to s}}{A_s} = \frac{A_f F_{fs} E_{b,f}}{A_s} = \frac{A_f F_{fs} \sigma T_f^4}{A_s} \quad (2)$$

where $A_f = \pi D_f^2 / 4$ and $A_s = \pi D_s^2 / 4$. The view factor between the furnace aperture and sample follows from the relation for coaxial parallel disks, Table 13.2,

$R_f = r_f / L_{sf} = 0.0125 \text{ m} / 0.750 \text{ m} = 0.01667$ $\qquad$ $R_s = r_s / L_{sf} = 0.0100 \text{ m} / 0.750 \text{ m} = 0.01333$

$$S = 1 + \frac{1 + R_s^2}{R_f^2} = 1 + \frac{1 + 0.01333^2}{0.01667^2} = 3600.2$$

Continued …..

$$F_{sf} = 0.5\left\{S - \left[S^2 - 4(r_s / r_f)^2\right]^{1/2}\right\} = 0.5\left\{3600 - \left[3600^2 - 4(0.05/0.0625)^2\right]^{1/2}\right\} = 0.000178$$

Hence the irradiation from the furnace is

$$G_f = \frac{\pi(0.025\ \text{m})^2/4 \times 0.000178 \times 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (3000\ \text{K})^4}{\pi\left(0.020^2\ \text{m}^2/4\right)} = 1277\ \text{W/m}^2$$

The irradiation from the surroundings which are large compared to the sample is

$$G_{sur} = \sigma T_{sur}^4 = 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}(300\text{K})^4 = 459\ \text{W/m}^2$$

Emissivity of the Sample: The total hemispherical emissivity in terms of the spectral distribution can be written following Eq. 12.38 and Eq. 12.30,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_s)\,d\lambda / \sigma T^4 = \varepsilon_1 F_{(0-\lambda_1 T_s)} + \varepsilon_2\left[1 - F_{(0-\lambda_1 T_s)}\right]$$

$$\varepsilon = 0.8\times0.066728 + 0.2[1 - 0.066728] = 0.240$$

where, from Table 12.1, with $\lambda_1 T_s = 4\ \mu\text{m}\times500\ \text{K} = 2000\ \mu\text{m}\cdot\text{K}$, $F_{(0-\lambda T)} = 0.066728$.

Absorptivity of the Sample: The total hemispherical absorptivity due to irradiation from the furnace follows from Eq. 12.46,

$$\alpha_f = \varepsilon_1 F_{(0-\lambda_1 T_f)} + \varepsilon_2\left[1 - F_{(0-\lambda_1 T_f)}\right] = 0.8\times0.945098 + 0.2[1 - 0.945098] = 0.767$$

where, from Table 12.1, with $\lambda_1 T_f = 4\,\mu\text{m}\times3000\ \text{K} = 12,000\ \mu\text{m}\cdot\text{K}$, $F_{(0-\lambda T)} = 0.945098.$ The total hemispherical absorptivity due to irradiation from the surroundings is

$$\alpha_{sur} = \varepsilon_1 F_{(0-\lambda_1 T_{sur})} + \varepsilon_2\left[1 - F_{(0-\lambda_1 T_{sur})}\right] = 0.8\times0.00234 + 0.2[1 - 0.002134] = 0.201$$

where, from Table 12.1, with $\lambda_1 T_{sur} = 4\ \mu\text{m}\times300\ \text{K} = 1200\ \mu\text{m}\cdot\text{K}$, $F_{(0-\lambda T)} = 0.002134$.

Evaluating the Energy Balance: Substituting numerical values into Eq. (1),

$$P_e = \left[+20\ \text{W/m}^2\cdot\text{K}(500-300)\text{K} - 0.767\times1277\ \text{W/m}^2\right.$$

$$\left. -0.201\times459\ \text{W/m}^2 + 0.240\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4(500\ \text{K})^4\right]\pi(0.020\ \text{m})^2/4$$

$$P_e = 1.256\ \text{W} - 0.308\ \text{W} - 0.029\ \text{W} + 0.267\ \text{W} = 1.19\ \text{W}$$ <

(b) The radiant power leaving the sample which is incident on the detector and within the spectral region, Δλ = 3 to 5μm, follows from Eq. 12.5 with Eq. 12.30,

$$q_{s-d,\Delta\lambda} = \left[E_{s,\Delta\lambda} + G_{f,ref,\Delta\lambda} + G_{sur,ref,\Delta\lambda}\right](1/\pi)A_s\cos\theta_s\cdot A_d\cos\theta_d / L_{sd}^2$$

where $\theta_s = 45°$ and $\theta_d = 0°$. The *emitted* component is

$$E_{s,\Delta\lambda} = \int_3^{5\mu m} \varepsilon_{\lambda,b} E_{\lambda,b}(T_s)$$

$$E_{s,\Delta\lambda} = \left\{\varepsilon_1\left[F_{(0-4\mu m,T_s)} - F_{(0-3\mu m,T_s)}\right] + \varepsilon_2\left[F_{(0-5\mu m,T_s)} - F_{(0-4\mu m,T_s)}\right]\right\}\sigma T_s^4$$

Continued …..

$$E_{s,\Delta\lambda} = \{0.8[0.066728 - 0.013754] + 0.2[0.16169 - 0.066728]\}\sigma(500\text{K})^4 = 217.5\ \text{W/m}^2$$

where, from Table 12.1, $F_{(0-3\mu m,T_s)} = 0.013754$ at $\lambda T = 3\,\mu m \times 500\ K = 1500\ \mu m\cdot K$; $F_{(0-4\mu m,T_s)} = 0.066728$ at $\lambda = 4\,\mu m \times 500\ K = 2000\ \mu m\cdot K$; and $F_{(0-5\mu m,T_s)} = 0.16169$ at $\lambda T = 5\mu m \times 500\ K = 2500\ \mu m\cdot K$.

The *reflected irradiation from the furnace* component is

$$G_{f,ref,\Delta\lambda} = \int_3^{5\mu m} (1-\varepsilon_\lambda) G_{f,\lambda} d\lambda$$

where $G_{f,\lambda} \approx E_{\lambda,b}(T_f)$, using band emission factors,

$$G_{f,ref,\Delta\lambda} = \{(1-\varepsilon)[F_{(0-4\mu m,T_f)} - F_{(0-3\mu m,T_f)}] + (1-\varepsilon_2)[F_{(0-5\mu m,T_f)} - F_{(0-4\mu m,T_f)}]\}G_f$$

$$G_{f,ref,\Delta\lambda} = \{0.2[0.9451 - 0.8900] + 0.8[0.9700 - 0.9451]\}1277\ \text{W/m}^2 = 39.51\ \text{W/m}^2$$

where, from Table 12.1, $F_{(0-3\mu m,T_f)} = 0.8900$ at $\lambda T_f = 3\ \mu m \times 3000\ K = 9000\ \mu m\cdot K$; $F_{(0-4\mu m,T_f)} = 0.9451$ at $\lambda T_f = 4\ \mu m \times 3000\ K = 12{,}000\ \mu m\cdot K$; and, $F_{(0-5\mu m,T_f)} = 0.9700$ at $\lambda T_f = 5\ \mu m \times 3000\ K = 15{,}000\ \mu m\cdot K$.

The *reflected irradiation from the surroundings* component is

$$G_{sur,ref,\Delta\lambda} = \int_3^{5\mu m} (1-\varepsilon_\lambda) G_{ref,\lambda} d\lambda$$

where $G_{ref,\lambda} \approx E_\lambda(T_{sur})$, using band emission factors,

$$G_{sur,ref,\Delta\lambda} = \{(1-\varepsilon_1)[F_{(0-4\mu m,T_{sur})} - F_{(0-3\mu m,T_{sur})}] + (1-\varepsilon_2)[F_{(0-5\mu m,T_{sur})} - F_{(0-4\mu m,T_{sur})}]\}G_{sur}$$

$$G_{sur,ref,\Delta\lambda} = \{0.2[0.002134 - 0.0001685] - 0.8[0.013754 - 0.002134]\}459\ \text{W/m}^2 = 4.44\ \text{W/m}^2$$

where, from Table 12.1, $F_{(0-3\mu m,T_{sur})} = 0.0001685$ at $\lambda T_{sur} = 3\ \mu m \times\cdot 300\ K = 900\ \mu m\cdot K$; $F_{(0-4\mu m,T_{sur})} = 0.002134$ at $\lambda T_{sur} = 4\ \mu m \times 300\ K = 1200\ \mu m\cdot K$; and $F_{(0-5\mu m,T_{sur})} = 0.013754$ at $\lambda T_{sur} = 5\ \mu m \times 300\ K = 1500\ \mu m\cdot K$. Returning to Eq. (3), find

$$q_{sd,\Delta\lambda} = [217.5 + 39.51 + 4.44]\ \text{W/m}^2 (1/\pi)[8\pi(0.020\ \text{m})^2/4]$$
$$\cos 45° \times 8\times10^{-5}\ \text{m}^2 \times \cos 0°/(1\ \text{m})^2 = 1.48\ \mu W \qquad <$$

COMMENTS: (1) Note that F_{fs} is small, since A_f, $A_s << L_{sf}^2$. As such, we could have evaluated $q_{f\to s}$ using Eq. 12.5 and found

$$G_f = \frac{E_{b,f}/\pi A_f (A_s/L_{sf}^2)}{A_s} = 1276\ \text{W/m}^2$$

(2) Recognize in the analysis for part (b), Eq. (3), the role of the band emission factors in calculating the fraction of total radiant power for the emitted and reflected irradiation components.